ELASTIC FORCE DEMANDS FROM BI-DIRECTIONAL EXCITATION

Dionisio Bernal

Civil and Environmental Engineering Department, Center for Digital Signal Processing,
Northeastern University, Boston, MA

Abstract

When the design response spectra for the two horizontal components of motion are of equal intensity the expectation of the peak seismic demand is given by the SRSS combination of the uni-directional responses. The variance of this response depends, however, on the probability densities of the cross-correlations and thus the upper bounds on the ratios of response to design level in elements that have important contributions from both loading directions are larger than in those dominated by uni-directional motion. This paper operates on the premise that it is desirable to equalize these bounds and attempts to do so by specifying the cross-correlations at a probability of exceedance that attains the objective. Derivation of the *pdf* of the cross correlation is required and is found that for closely spaced frequencies it is well approximated by the *pdf* of the unlagged coherency times the standard correlation coefficient. The *pdf* of the unlagged coherency, in turn, is shown to be well approximated by a shifted and scaled beta distribution with parameters $\alpha = \beta = 2.55$. It is shown that when the results obtained are translated into the 100%+X% combinations rule the consistent value of X is sixty.

Introduction

Combination of modal responses to a uni-directional input is a classical subject in earthquake engineering. The SRSS rule, which applies to stationary uncorrelated responses, appeared first in the Ph.D thesis of E. Rosenblueth (1951) and subsequently in a paper by Goodman, Rosenblueth and Newmark (1953). The pioneering contribution on the rules to combine correlated responses is presented in Rosenblueth and Elorduy (1969) with formulas later developed by Der Kiureghian (1979) now widely adopted under the CQC designation. This paper is concerned with the combination of modal responses to orthogonal input components with attention limited to responses to excitations in the horizontal plane. The pioneering contribution here is from Rosenblueth and Contreras (1977) who proposed, based on analytical considerations and some simplifications, the now popular 30% rule.

The combination of modal responses from multiple inputs involves cross-correlation coefficients that depend on the coherency between components and have symmetric *pdfs* when the motions are specified in so-called principal directions. A proposal that the major principal direction at a site is aligned with a line joining the epicenter with the site was made (based on the analysis of five records) by Penzien and Takizawa (1975) but this contention is not supported by results obtained here using 40 pairs of bi-directional records from the CSMIP database. What examination shows is that the directions for which the correlation between the records is zero fluctuates notably when computed in a moving window (of 4 to 6 seconds size), suggesting that there is limited usefulness in the principal direction concept. We note that fluctuation in the computed principal directions is a byproduct of the fact that the seismic intensity in any two horizontal directions is not too dissimilar in most records, making the data "quasi-circular" and the principal orientations easily shift as a the computation window moves along. Whether justified on grounds that the spectra are equal, or in any other way, the bottom line is that seismic provisions are based on the premise that any two orthogonal directions in the horizontal plane can be treated as principal directions. From the perspective of elastic force demand computation the implication is that the cross-correlation coefficients that enter in the estimation of the expected value of the peak response are zero.

When one looks at the variance of the peak response, however, the probability density of the cross-correlation enters the picture and the relevant observation, from a design perspective, is that response quantities that are notably affected by bi-directional input may have significantly larger variance than those primarily dependent on one input component. Bounds on the limit by which demands may exceed the design estimate, therefore, are larger in bi-directional sensitive quantities than in unidirectional controlled ones. We note, for clarity, that while any element can be considered as affected by both horizontal components (since axes can be rotated at will) the opposite is not true. Namely, there are elements whose forces are significantly affected by both direction of loading for any orientation of the analysis axes. In this regard note that the typical selection of building axes can be viewed as the one that maximizes the number of response quantities that depend on one input or the other. The purpose of this paper is to determine if the mentioned increase in variance is potentially relevant in design and, if it is, to suggest ways to account for it properly.

Worth noting from the outset is the fact that the *pdf* of correlations and cross correlations are affected differently by the eigenvalue gap. In particular, as two eigenvalues approach each other the correlation between the modal responses approaches unity and, as a consequence, the variance of the realizations approaches zero. The variance of the correlation between closely space modes is, therefore, small. In contrast, in the cross-correlation case the expected value of the distribution is near zero, independent of the frequency ratio, but the variance increases as modal separation decreases. In this paper we pursue consideration of the high variance issue by specifying the cross correlation coefficients at a probability of exceedance smaller than the mean.

An outline of the paper is as follows: it is first shown that for zero eigenvalue gap the *pdf* of the cross correlation is well approximated by the *pdf* of the unlagged coherency and a model for this function is derived using results from a large ensemble of synthetic bi-directional records generated using the Rezaeian Der Kiureghian (2012) ground motion model. Reduction of the cross correlation with frequency separation is shown to be much slower than that of the correlation and it is argued that this results from the difference between the Power Spectral Density (PSD) , which is real and determines correlations, and the coherency, which is complex and determines cross correlations. An explicit formula for reduction of the cross-correlation with frequency gap, however, is not presented. The analytical part of the paper closes with an analysis that translates the results obtained into the 100 +X% rule and a brief concluding section closes the paper.

Principal Ground Motion Directions

The subject of principal directions shows up when bi-directional excitation is considered because these directions specify the orientation for which the coherency between the orthogonal components is zero. Equivalently, the principal directions are also the directions for which the empirical covariance is diagonal. Namely, if $\ddot{X} \in R^{3xN}$ is a matrix that lists the ground motion at a point in any three orthogonal directions and $\ddot{X}_{(n_1,n_2)} = \ddot{X}(:,\mathrm{n}_1:\mathrm{n}_2)$ where n_1 and n_2 are indices selected to clip and internal portion, the empirical covariance for the clipped segment is

$$Q_{(n_1,n_2)} = \frac{1}{n_2 - n_1 + 1} X_{(n_1,n_2)} \cdot X^T_{(n_1,n_2)} \tag{1}$$

where N is the total number of time stations. Analysis shows that one of the principal directions is vertical (or nearly so) so the other two are in the horizontal plane. The first to indicate that principal directions could be defined for seismic records was A. Arias (1970), although reference to the matter is typically misplaced to a publication by Penzien and Watanabe (1975) who apparently were unware of Arias work at the time of writing. Arias did not pursue the principal directions topic but Penzien and Watanabe (1975) made the claim that these directions are reasonably stationary during the strong motion and are approximately aligned with a line connecting the epicenter with the site. Although several writers have used this model (Yeh and Wen 1990; Kubo and Penzien 1979; Heredia-Zavoni and Machicao-Barrionuevo 2004; Menun and Der Kiureghian 1998a, 1998b, 2000; Rezaeian and Der Kiureghian 2010, 2012) confirmation of these claims could not be found in the literature and an examination carried out here using 40 bi-directional records did not support either contention. As noted first in the introduction, the lack of stationarity is easily rationalized by recognizing that principal directions are the axes of an ellipse that essentially contains the data considered and since the dimensions of the ellipse principal axis are not too different, rotations are easily realized. Fig.1 plots data for a 4 sec window during the strong motion at CSMIP station 14311 during the Whittier earthquake and shows the quasi-circular nature of the data while Fig.2 shows the rotation of the axes in two nearby windows.

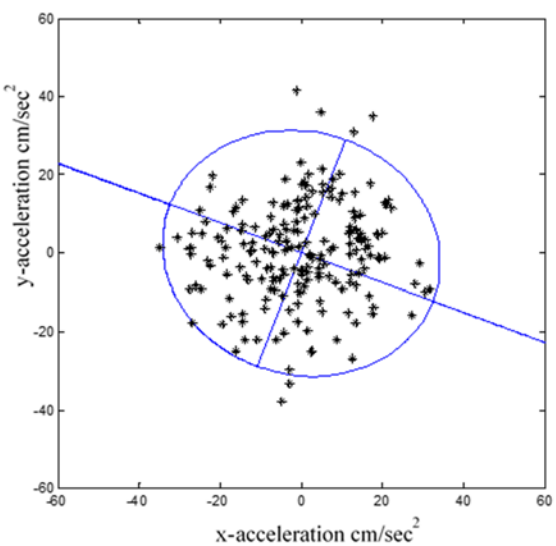

Fig.1. Accelerations, principal directions and inscribing ellipse for the first 4 sec window in the strong motion defined using the $t_{0.9}$ Arias Intensity.

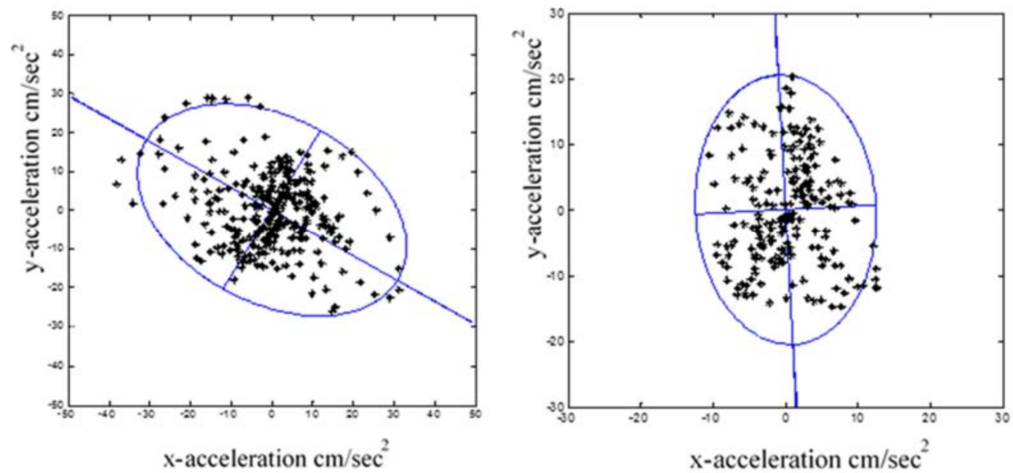

Fig.2. Accelerations, principal directions and inscribing ellipses a) window from 6-10 sec b) window from 12-16 sec (same records as Fig.1)

Directivity

A convenient parameter to characterize the shape of the "best fit" ellipse, which we define here as "directivity", is

$$\gamma = 1 - \sqrt{\frac{s_2}{s_1}} \tag{2}$$

where s_1, and s_2 are the largest and the smallest singular values of the empirical covariance. For a circular shape $\gamma = 0$ and for highly elongated shapes γ approaches 1. The term directivity is also used as a qualitative term to indicate the focusing of wave energy along the fault in the direction of rupture but risk of confusion with the quantitative term in eq.2 does not appear significant. Directivities computed for 40 bidirectional ground motions taken from the CSMIP site proved to have a mean of about 0.3 (Gali, 2015), indicating (again) that the "principal directions" are unlikely to be stationary, some examples of the evolution of the mayor principal direction computed on a 4 second moving window are depicted in Fig.3. We close this section by noting that although the literature is replete with papers that use the concept of ground motion principal directions there is also wide spread recognition that it is reasonable to assume that the motion in any two orthogonal horizontal directions can be treated as uncorrelated, i.e., that any two orthogonal directions in the horizontal plane can be treated as principal.

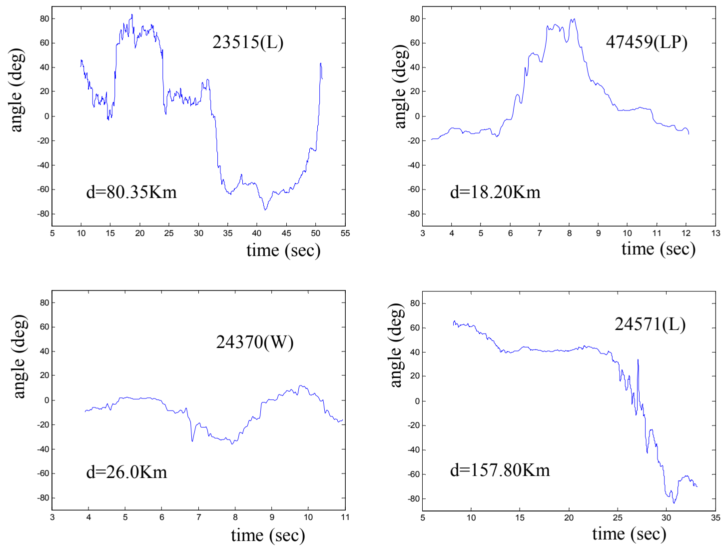

Fig.3. Evolution of major principal direction for four CSMIP stations -distances are from the site to the epicenter and letter in parenthesis designates the earthquake.

Cross-Correlation between Modal Responses

The cross-correlation coefficient between the responses of two modes, *i* and *j* to ground motion components k and ℓ is the expected value of a distribution and writes (Der Kiureghian and Neuhoffer 1991)

$$\rho_{s_{ki},s_{\ell j}} = \frac{1}{\sigma_{ski}\sigma_{s\ell j}} \int_{-\infty}^{\infty} h_i(\omega)\cdot h_j(-\omega)\cdot g_{k,\ell}(\omega)\cdot d\omega \tag{3}$$

with

$$\sigma^2_{s_{a,b}} = \int_{-\infty}^{\infty} h_b(\omega)\cdot h_b(-\omega)\cdot g_{\mathrm{a,a}}(\omega)\cdot d\omega \quad with \quad a=k,\ell \quad b=i,j \tag{4}$$

where $h_b(\omega)$ is the relative displacement transfer function

$$h_b(\omega) = \frac{1}{(\overline{\omega}_b^2 - \omega^2) + 2\overline{\omega}_b\omega\xi_b i} \tag{5}$$

with $\overline{\omega}_b, \xi_b$ as the undamped natural frequency and modal damping ratio and $g_{k,\ell}(\omega)$ is the cross spectral density (CSD) between the *k* and the ℓ excitation components. This last function is the Fourier transform of the cross correlation between the signals and thus

$$g_{k,\ell}(\omega) = \int_{-\infty}^{\infty} g_{k,\ell}(\tau)\cdot e^{-i\omega\tau} d\tau \tag{6}$$

where

$$g_{k,\ell}(\tau) = E\left(\ddot{u}_k(t) \cdot \ddot{u}_\ell(t+\tau)\right) \tag{7}$$

The CSD is typically specified in terms of the coherency function, $\gamma_{k,\ell}(\omega)$ and the PSD functions as

$$g_{k,\ell}(\omega) = \gamma_{k,\ell}(\omega) \cdot \sqrt{g_{k,k}(\omega) \cdot \mathrm{g}_{\ell,\ell}(\omega)} \tag{8}$$

Assuming that the PSD functions for both components are equal and vary slowly with frequency one can treat them as a constant without incurring undue error and one can write

$$\rho_{s_{ki},s_{\ell j}} = \int_{-\infty}^{\infty} \bar{h}_i(\omega) \cdot \bar{h}_j(-\omega) \cdot \gamma_{k,\ell}(\omega) \cdot d\omega \tag{9}$$

where the transfer functions have been normalized such that

$$\int_{-\infty}^{\infty} \left|h_b(\omega)\right|^2 d\omega = 1 \quad for \quad b = i, j \tag{10}$$

Realizations

Let a pair of bi-directional components be referred to by the index q and let the coherency function for a given pair be $z_q(\omega)$. The realized value of the cross correlation is then

$$p_q = \int_{-\infty}^{\infty} \bar{h}_i(\omega) \cdot \bar{h}_j(-\omega) \cdot z_q(\omega) \cdot d\omega \tag{11}$$

and it follows that eq.3 is the mean of the realizations given by eq.11. If one performs a Monte Carlo study that generates values of p_q the distribution of this random variable can be estimated and used to specify the cross correlation at whatever probability of being exceeded one selects.

Zero Eigenvalue Gap

In this case eq.11 reduces to

$$p_q = \int_{-\infty}^{\infty} \left|\bar{h}_i(\omega)\right|^2 \cdot z_q(\omega) \cdot d\omega \tag{12}$$

and if the damping is low, as is typically the case, and the real part of the function $z_q(\omega)$, known as the unlagged coherency, is reasonably flat in the vicinity of the natural frequency, then with good approximation one has

$$p_q \cong \Re\left(z_q(\bar{\omega}_b)\right) \tag{13}$$

Namely, the cross-correlation in this case is well approximated by the unlagged coherency evaluated at the undamped natural frequency of the mode.

Finite Eigenvalue Gap

The finite eigenvalue gap case is more difficult to evaluate because the product of the two transfer functions has real and imaginary parts and the real and the imaginary parts of the coherency enter the formulation. It appears, however, that for small eigenvalue gaps the cross correlation can be approximated as the value for zero gap times the CQC correlation. To illustrate assume the unlagged coherency is sufficiently flat in the region where the transfer function product has important values, one then has

$$p_q = \Re\left(z_q(\hat{\omega})\right) \cdot \int_{-\infty}^{\infty} \bar{h}_i(\omega) \cdot \bar{h}_j(-\omega) \cdot d\omega + \chi \tag{14}$$

where χ is the contribution that comes from the integration of the product of the imaginary part of the coherency times the imaginary part of the product of the transfer functions. Noting that the integral in eq.14 is the CQC correlation coefficient (Der Kiureghian 1979) one has that for values of r near unity

$$p_q \cong \Re\left(z_q(\hat{\omega})\right) \cdot \rho_{CQC} \tag{15}$$

with

$$\rho_{CQC} = \frac{8\xi^2(1+r) \cdot r^{1.5}}{\left(1-r^2\right)^2 + 4\xi^2 r\left(1+r^2\right) + 8\xi^2 r^2} \tag{16}$$

where r is the ratio of the modal frequencies.

Unlagged Coherency

Several models for the expectation of the coherency as a function of distance and frequency have been proposed and a good review can be found in Zerva and Zervas (2002). What is needed for the present application, however, is the probability distribution of the unlagged coherency for orthogonal motions at a point in the horizontal plane. For this distribution no models could be identified so it became necessary to develop one. One possibility was to work with an ensemble of real bi-directional records but we opted for synthetic motions to allow a more convenient examination of the effect of the key motion parameters.

The bi-directional ground motion model selected is the one developed in Rezaeian and Der Kiureghian (2010, 2012). This model generates synthetic earthquake-like signals by filtering white noise through a time dependent impulse response function that is modulated to incorporate the evolution of the ground motion intensity. The model has six parameters *for each direction* of motion, the Arias intensity, I_a, the Arias effective duration taken as the 5-95 interval of the evolutionary intensity, $D_{5\text{-}95}$, the central value, and rate of change of the evolutionary frequency, ω_{mid} and ω', the critical damping ratio of the filter, ξ, and the time at which the central frequency of the impulse is realized, t_{mid}. These six parameters have probability distributions derived from data that are conditional on four parameters: the type of faulting, F, equal to zero for strike-slip and one for reverse faulting, the magnitude, $M \geq 6$, the epicentral distance R, $10 \leq R \leq 100$ (in Km) and the shear wave velocity in the top 30m V_{S30}, with $V_{s30} \geq 60\, m/sec$. The model considers the correlation between the parameters in the two horizontal directions by prescribing a correlation matrix derived from real data.

While derivation of a *pdf* for the unlagged coherency that is a function of all the parameters of the ground motion model is possible the level of complexity would not be useful for the purposes of specifying the cross correlation. The approach we select here is to examine how the unlagged coherency depend on each of the motion parameters individually and then, in light of the results obtained select a set of parameters for which to derive a single *pdf*.

Type of Faulting

The standard deviation of the unlagged coherency vs frequency at 40Km for V_{s30} = 500 m/sec and M = 7.5 based on 50 simulations is depicted for the two types of faulting in Fig.4. As can be seen, there is no clear indication that the unlagged coherency is larger for either type of faulting so we select the strike-slip model for specificity.

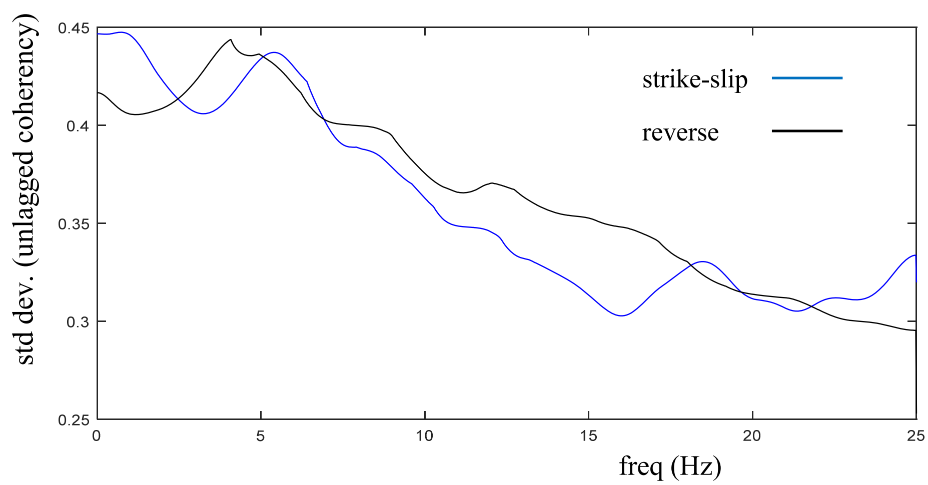

Fig.4. Standard deviation of the unlagged coherency vs frequency, M = 7.5, R = 40Km, V_{S30}=500 m/sec.

Distance to Epicenter

Plots of the standard deviation of the unlagged coherency for three different distances to the epicenter are depicted in Fig.5. The results suggest that the function decreases with distance at the higher frequencies but that in the in the most important range, i.e. up to 10Hz or so there is not much difference or a clear pattern so we take, for the Monte Carlo study, R = 40 Km.

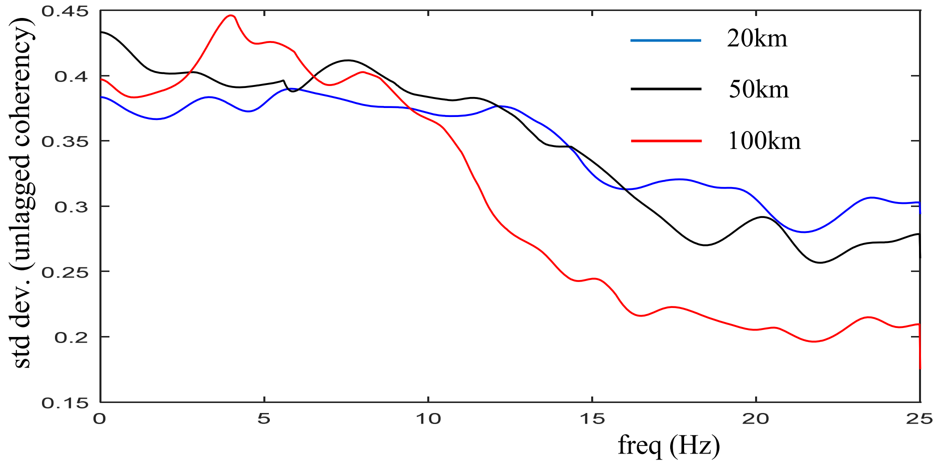

Fig.5. Standard deviation of the unlagged coherency vs frequency, M = 7.5, Strike-Slip, V_{S30}=500 m/sec.

Magnitude

Results illustrating the effect of magnitude are depicted in Fig.6. As can be seen, the curves are not ordered by magnitude in any particular pattern and for the lower frequencies the difference between the results is particularly small. In the Monte Carlo study we operate with M = 7.0.

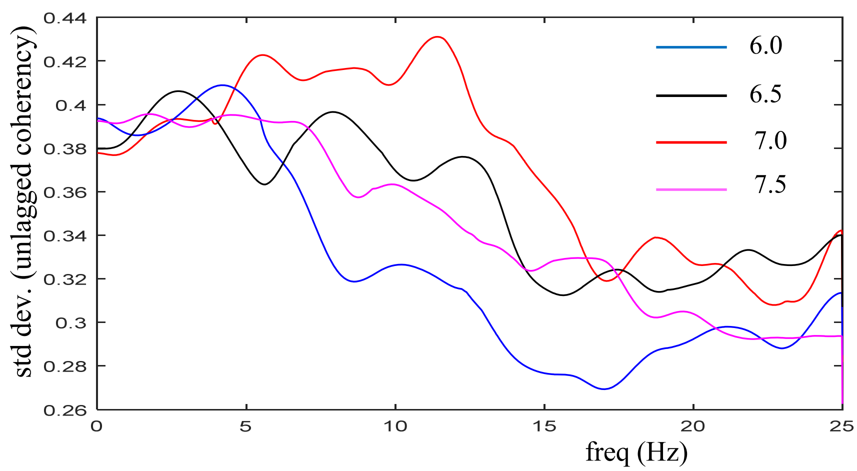

Fig.6. Standard deviation of the unlagged coherency vs frequency, R = 40 Km, Strike-Slip, V_{S30}=500 m/sec.

Shear Wave Velocity

Results plotted in Fig.7 for three values of the shear velocity, which correspond to very soft soil, dense soils and rock, show that the unlagged coherency is larger in the two stiffer soils than in the very soft one, although the difference between the dense soil and the rock situation is not systematic or significant. In the Monte Carlo study we operate with V_{S30} = 750 m/sec

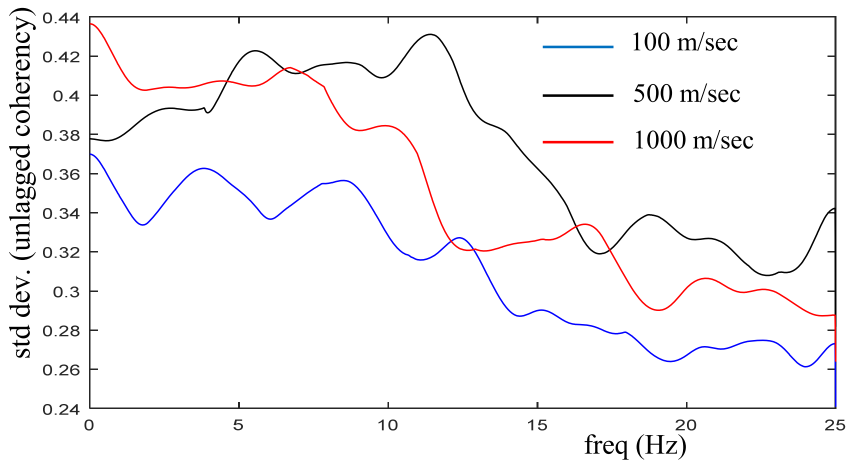

Fig.7. Standard deviation of the unlagged coherency vs frequency, R = 40 Km, Strike-Slip, M = 7.

<u>Probability Density Estimation</u>

On light of the results of the previous examinations we estimate the density of the unlagged coherency based on 500 simulations using the parameters {F=0, M = 7, R = 40 km, V_{S30} = 750m/sec) and plot the results in the same format used previously in Fig.8. As can be seen, the variation of the standard deviation with frequency is small.

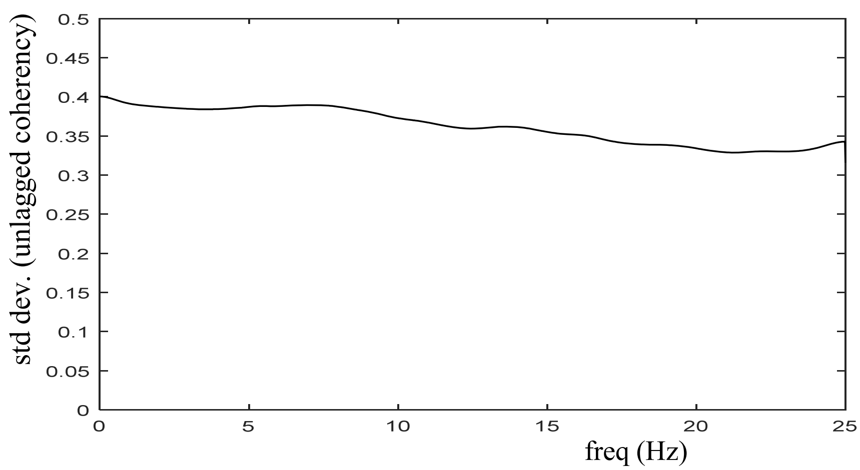

Fig.8. Standard deviation of the unlagged coherency vs frequency based on 500 simulated bi-directional records obtained for the parameters described in the text.

The normalized histograms of the unlagged coherency at three different frequencies are depicted in Fig.9 and a plot of the values at three different probability of exceedance for the full frequency range considered is depicted in Fig.10.

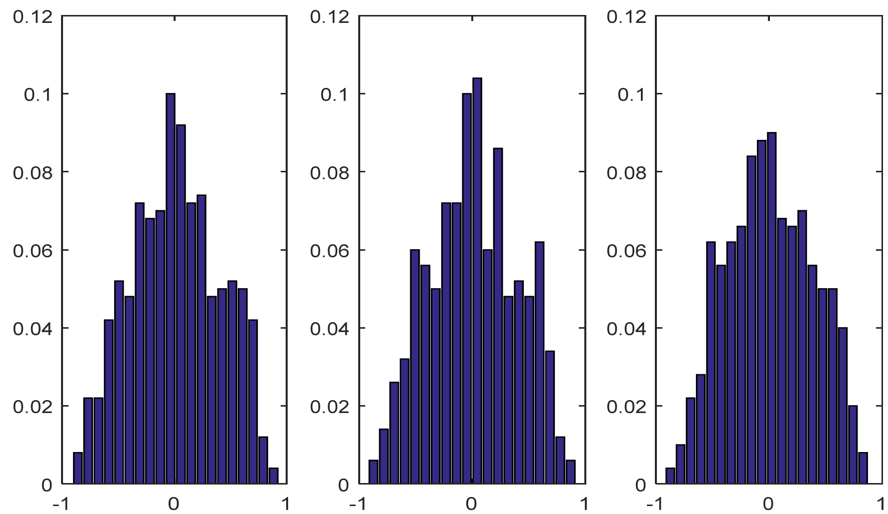

Fig.9. Unlagged coherency vs frequency based on 500 simulated bi-directional records obtained for the parameters described in the text.

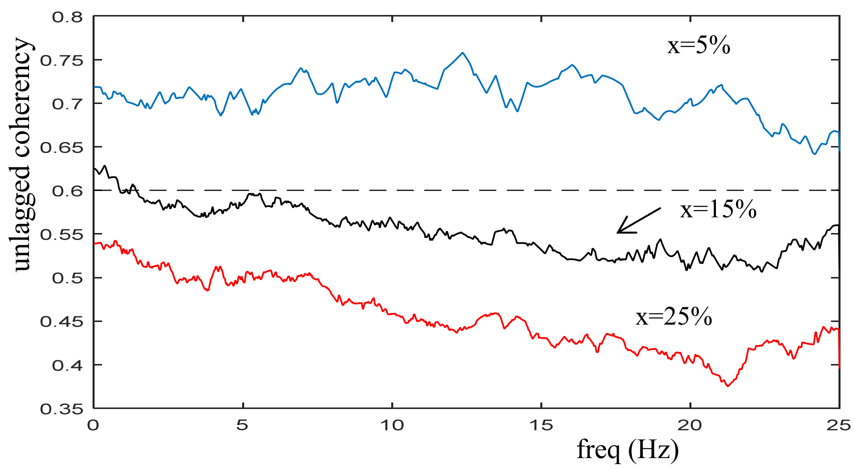

Fig.10. Unlagged coherency vs frequency at 3 different probability of exceedance.

We postulate a shifted scaled symmetric beta function for the *pdf* of the unlagged coherency. Since we have three constraints from the *cdf* in Fig.10 and there is only one free parameter (as $\alpha = \beta$) we select the distribution parameter in a least square sense. Given that the changes in the values in Fig.10 are small for small frequencies and that simplicity is of the essence we opt for a frequency independent *pdf*. Taking the unlagged coherence critical values at {0.72, 0.60 and 0.52} for probabilities of exceedance of 5, 15 and 25% respectively one finds that $\alpha = \beta = 2.55$ provides a good fit, with the theoretical values being {0.75, 0.6 and 0.5}. In the treatment that follows we select the unlagged coherency from the distribution at the 15% probability of exceedance and thus take it equal to 0.6. While we could have selected this value without fitting the analytic *pdf* to the data the *pdf* allows easy selection of the unlagged coherency in case a different probability of exceedance is of interest in some future work.

Response Spectrum Estimation of Maximum Response

For structures on a rigid base subjected to bi-directional excitation the expected value of the maximum response can be written as (Der Kiureghian and Neunhofer 1991)

$$E[\max|z(t)|] = \left(\sum_{k=1}^{2} \sum_{\ell=1}^{2} \sum_{i=1}^{n} \sum_{j=1}^{n} b_{ki} b_{\ell j} \rho_{ski,s\ell j} D_k(\omega_i, \xi_i) D_\ell(\omega_j, \xi_j) \right)^{\frac{1}{2}} \tag{17}$$

where $D_k(\omega_p, \xi_p)$ is the relative displacement spectrum for the *k* motion, b_{ki} is the contribution of the i^{th} mode to the quantity being evaluated due to the earthquake in the *k* direction for a unit value of the displacement spectrum and $\rho_{ski,s\ell j}$ is the correlation between the responses of modes *i* and *j* for input directions *k* and ℓ. The result in eq.17 can be written in convenient matrix form as

$$E[\max|z(t)|] = \left(m_t^T \Gamma m_t + m_x^T \Upsilon m_y \right)^{\frac{1}{2}} \tag{18}$$

where $m_t = m_x + m_y$ with $m_x, m_y \in R^{nx1}$ are the vectors that contain the modal contributions for the input in the x-x and y-y directions, Γ is the matrix of correlation coefficients and Υ is the matrix of cross correlations. In the present seismic guidelines the second term in eq.18 is taken as zero and thus the response is taken as

$$E[\max|z(t)|] = \left(m_x^T \Gamma m_x + m_y^T \Gamma m_y \right)^{\frac{1}{2}} \tag{19}$$

which is the SRSS of the correlated responses to one directional input. The estimation of the peak response based on the work presented previously (neglecting reductions in the correlation with eigenvalue gap) is

$$\max|z(t)| = \left(m_t^T \Gamma m_t \mp 1.2\, m_x^T m_y \right)^{\frac{1}{2}} \tag{20}$$

where the expectation symbol has been removed because eq.22 is no longer (strictly) and expectation estimate.

On the 100% +X% Rule

Consider a quantity y(t) whose contribution from excitation in the 1-1 direction is $y_1(t)$ and from the 2-2 direction $y_2(t)$. If the two time histories are uncorrelated in the particular realization considered then

$$[\max(y)]^2 = \max(y_1^2) + \max(y_2^2) \tag{21}$$

We take $\max(y_2^2) = \beta^2 \max(y_1^2)$ and, without loss of generality assume that y_1 is the response with the largest absolute value so $\beta < 1$ and get

$$|y|_{\max} = |y_1|\sqrt{1+\beta^2} \tag{22}$$

In the 100+ X% combinations rule the estimated maximum response is taken as

$$|y|_{\max} = |y_1| + X\beta|y_1| \tag{23}$$

and it thus follows, equating eqs.22 and 23 and taking X = 0.3 than $\beta = 0.659$. In other words, the SRSS rule and the 30% combination coincide if the smaller of the two responses is around 2/3 of the largest. Assume know that the value of the cross-correlation realized in the response is ρ. Following the same analysis as before one concludes that the maximum response is now

$$|y|_{\max} = |y_1|\sqrt{1+\beta^2+2\beta\rho} \tag{24}$$

and equating this with eq.23 gives

$$\sqrt{1+\beta^2+2\beta\rho} = 1 + X\beta \tag{25}$$

or

$$X = \frac{-1+\sqrt{1+\beta^2+2\beta\rho}}{\beta} \tag{26}$$

The value of X from eq.26 plotted vs. ρ for $\beta = 0.659$ is depicted in Fig.11.

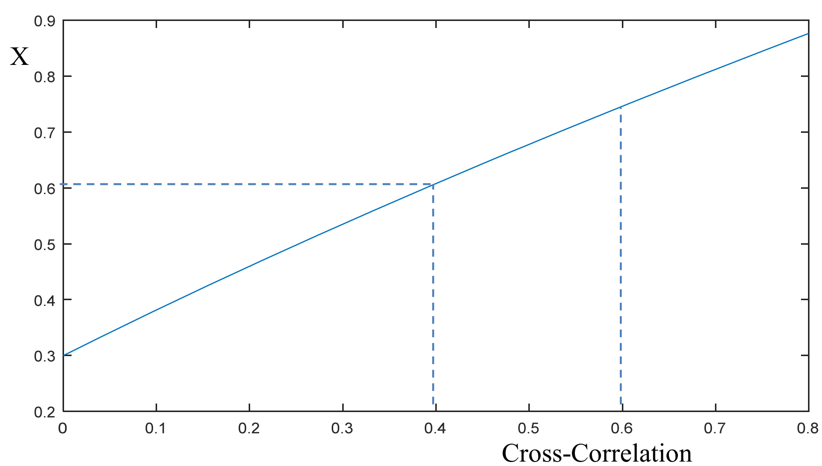

Fig.11. Value of X in the 100+X% rule for which the maximum response is "correctly" estimated vs the cross correlation between bi-direction responses.

If the responses in both directions were governed by modes with identical periods and one accepts the 15% probability of exceedance of the previous section for the cross-correlation one would select to “read” X from Fig.10 using 0.6 for the cross correlation. In most instances, however, the orthogonal responses will be governed by different modes and this selection is perhaps too conservative. Here we (heuristically) select a 50% reduction on the cross-correlation and thus read X at a cross-correlation of 0.4 obtaining the value of 0.6.

Numerical Validation

The previous section suggests a 60% rule. In this section we test this rule using two versions of a 10-story 3-D shear building having the irregular plan distribution shown in Fig.12. The building versions differ only on the rotational inertia of the floor plan and the amount of modal damping; in the case with the higher damping there are some modestly closed periods. The structure is excited with 100 simulated biaxial records and attention is focused on the maximum shear in the inclined wall element at the second floor. The following results are obtained for the two structures:

1) Exact maxima for each of the 100 bi-directional records.
2) Exact maxima for the y-y components of each of the records (single input response)
3) Response spectral solutions for each of the directions of analysis.

For the response spectrum solutions we use the exact spectral ordinates for each motion to ensure that all discrepancy with exact responses derives only from the modal combination. Inspection of numerical results shows that the differences between the CQC and SRSS prediction in the one-directional analyses are trivial.

Since a 60% rule is necessarily more conservative than the 30% one a comparison based on the degree of conservatism is not meaningful. A reasonable gauge, however, is a comparison between the most severe under-predictions when using any of the rules and those that occur in a one directional analysis. A reasonable way to inspect the results is to look at the mean of the largest “n” underestimated responses and compare them with the unidirectional result. The results are depicted in Figs.13 and 14 and show, as expected, that the 30% rule is un-conservative. The suggested 60% rule is slightly conservative in the 2% damped building and less so in the 5% damping case.

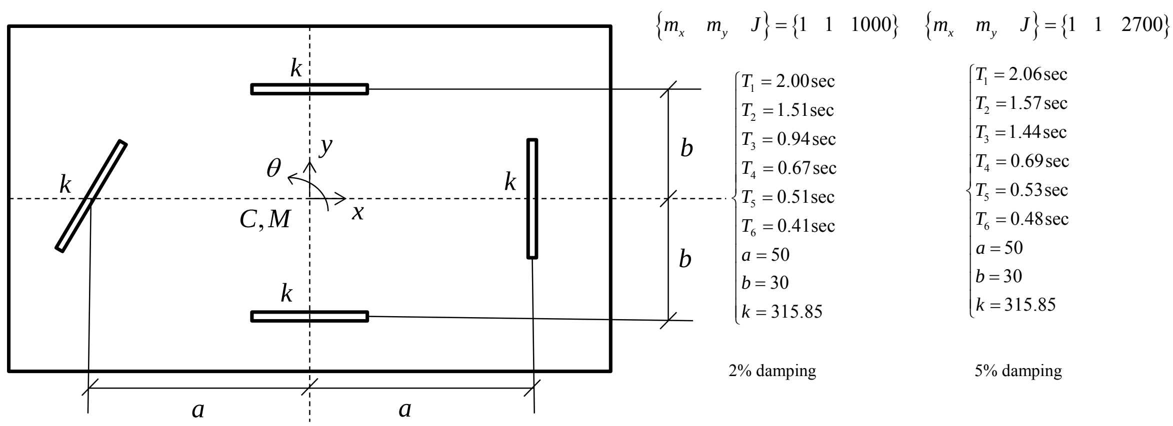

Fig.12. Plan view and periods of the 10 story shear buildings used in the validation study, damping is 2% in all the modes.

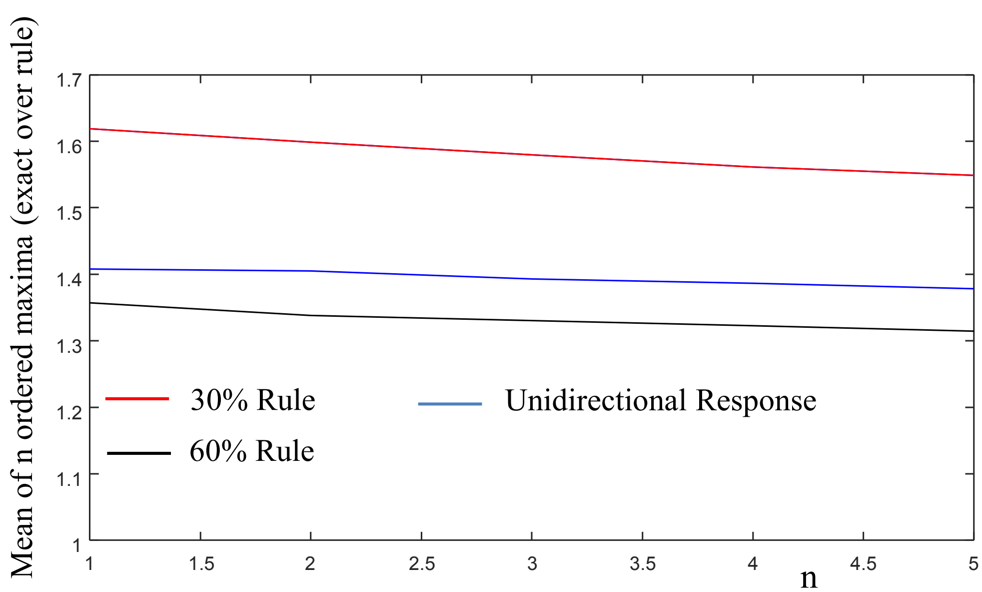

Fig.13. Mean of ordered maxima of the ratio of exact to estimated response based on 100 simulations for building of Fig.12with 2% damping.

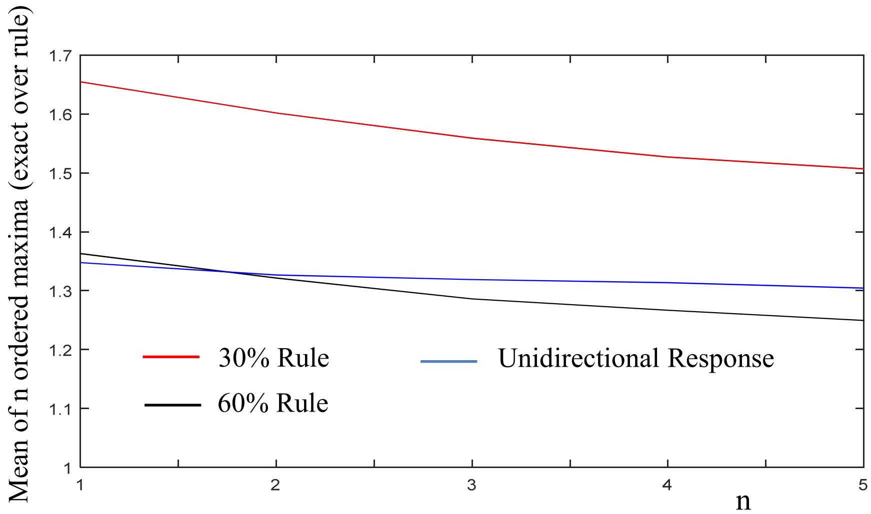

Fig.14. Mean of ordered maxima of the ratio of exact to estimated response based on 100 simulations for building of Fig.12 with 5% damping.

Concluding Comments

Response estimation is presently carried out at the level of expectation. Implicit in this approach is the assumption that variance related under-predictions are adequately accommodated by the design. Since the overwhelming majority of the accumulated numerical experience comes from studies with uni-directional excitation a question that arises is whether the larger variance in elements that are strongly affected by multiple components merits adjustments. On the premise that it is desirable to have the same level of protection against the largest possible underestimations in force demand one can state that the SRSS and the 30% combination rules are unconservative. On this matter the paper shows that similar performance is realized if the cross-correlation coefficient for zero eigenvalue gaps is taken as 0.60 and that, in the case of provisions based on the 100+X% rule, if X is taken around 60.

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