# EVALUATION OF NONLINEAR STATIC PROCEDURES USING STRONG-MOTION BUILDING RECORDS

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## Abstract

The objective of this investigation is to evaluate the FEMA-356 Nonlinear Static Procedure (NSP), the Sum-Difference procedure, and the Modal Pushover Analysis (MPA) procedure using recorded motions of buildings that were damaged during the 1994 Northridge earthquake. It is found the FEMA-356 NSP and the Sum-Difference procedures typically underestimates the drifts in upper stories and overestimates them in lower stories. The MPA procedure provides estimates of drifts that are better compared to the FEMA-356 NSP and the Sum-Difference procedure. In particular, the MPA procedure is able to capture the effects of higher modes.

## Introduction

Nonlinear static pushover (NSP) analysis is used commonly by the current civil engineering practice for estimating seismic demands at low performance levels, such as life safety and collapse prevention. In the NSP procedure (FEMA, 1997a; 1997b), the seismic demands are computed by nonlinear static analysis of the structure subjected to monotonically increasing lateral forces with an invariant height-wise distribution until a predetermined target displacement is reached. Both the force distribution and target displacement are based on the assumption that the response is controlled by the fundamental mode and that the mode shape remains unchanged after the structure yields. The underlying assumptions and limitations of the NSP analysis have been evaluated (Elnashai, 2001; Fajfar and Gaspersic, 1996; Gupta and Krawinkler, 1999; Maison and Bonowitz, 1999; Reinhorn, 1997; Skokan and Hart, 2000) and it has been found that satisfactory predictions of seismic demands are mostly restricted to low- and medium-rise structures for which higher mode effects are likely to be minimal and the inelastic action is distributed throughout the height of the structure (Krawinkler and Seneviratna, 1998).

Since the invariant force distributions cannot account for redistribution of inertia forces because of structural yielding and the associated changes in the vibration properties of the structure, NSP using adaptive force distributions that attempt to follow more closely the time-variant distributions of inertia forces have also been developed (Bracci et al., 1997; Gupta and Kunnath, 2000). The most recent version of the FEMA document (ASCE, 2000), denoted as FEMA-356, includes the option to consider such an adaptive distribution in addition to the invariant force distributions of the earlier version (FEMA, 1997b). While the adaptive force distributions may provide better estimates of seismic demands (Gupta and Kunnath, 2000), they are conceptually complicated, computationally demanding for routine application in structural engineering practice, and require special purpose computer program to carry out the step-by-step analysis.

Attempts have also been made to consider more than the fundamental vibration mode in pushover analysis. The Multi-Mode Pushover (MMP) procedure (Paret et al., 1996; Sasaki et al., 1998) provided information on possible failure mechanisms due to higher modes, which may be missed by the standard NSP analyses. But other information of interest in the design process, such as story drifts and plastic rotations, could not be computed by the MMP procedure. The Sum-Difference procedure (Kunnath and Gupta, 2000; Matsumori et al., 1999) also provided “useful” information but was tested on a single building (Kunnath and Gupta, 2000).

The modal pushover analysis (MPA) procedure, developed based on structural dynamics theory, attempted to capture contributions of several modes of vibration (Chopra and Goel, 2001; 2002). This procedure was systematically evaluated using six buildings (Goel and Chopra, 2004), and generic frames (Chopra and Chintanapakdee, 2004). It was found that with sufficient number of “modes” included, the height-wise distribution of story drifts estimated by MPA is generally similar to trends noted from nonlinear RHA. Furthermore, the additional error (or bias) in the MPA procedure applied to inelastic structures is small to modest compared to the bias in response spectrum analysis (RSA) applied to elastic structures – the standard analytical tool for the structural engineering profession – unless the building is deformed far into the inelastic region with significant stiffness and strength deterioration.

Most previous research on development and evaluation of the NSP and improved procedures is based on response of analytical models subjected to recorded and/or simulated earthquake ground motions. Recorded motions of buildings, especially those deformed into the inelastic range, provide a unique opportunity to evaluate such procedures. Therefore, the principal objective of this investigation is to evaluate the three NSP procedures – the FEMA-356 NSP, the MPA, and the Sum-Difference – using recorded motions of buildings that were deformed beyond the elastic limit during the 1994 Northridge earthquake. This paper summarizes the findings of this investigation; intermediate results from this investigation have also been reported in several previous publications (Goel, 2003a; 2003b; 2003c; 2004a; 2004b).

## Selected Buildings

Recorded motions of buildings that were deformed beyond the yield limit (or damaged) during the earthquake are required for this investigation. For this purpose, four buildings have been identified (Table 1) for which the motions were recorded during the 1994 Northridge earthquake. Of these four buildings, three buildings – Van Nuys 7-Story, Sherman Oaks 13-Story, and Los Angeles 19-Story – have been extensively instrumented by California Strong Motion Instrumentation Program (CSMIP) and the fourth – Woodland Hills 13-Story – has been nominally instrumented in accordance to the local code requirements.

The selected buildings have all been reported to be damage during the 1994 Northridge earthquake. Several columns between the fourth and fifth floor in the longitudinal frame on south side of the Van Nuys building failed in shear (Islam et al., 1998; Li and Jirsa, 1998; Naeim, 1997). The damage to Woodland Hills building consisted of local fracture at the beam-to-column welded joints (Uang et al., 1997). The Sherman Oaks building suffered cracks at many beam-column joints (Shakal et al., 1994). Finally, the Los Angeles building was reported to have suffered moderate damage in the form of buckling in some braced at upper floor levels (Naeim, 1997). Further details of these buildings and damage are available elsewhere (Goel, 2003a; 2003b; 2003c; 2004a; 2004b).

Table 1. Selected buildings, and peak ground and structure accelerations recorded during the 1994 Northridge earthquake.

| Buildings name | CSMIP Station | Number of Stories | Peak accelerations (g) | |
|---|---|---|---|---|
| | | | Ground | Structure |
| Van Nuys 7-Story Hotel | 24386 | 7 | 0.47 | 0.59 |
| Woodland Hills 13-Story | C246 | 12/2 | 0.44 | 0.33 |
| Sherman Oaks 13-Story | 24322 | 13/2 | 0.46 | 0.65 |
| Los Angeles 19-Story | 24643 | 19/4 | 0.32 | 0.65 |

## Analysis of Recorded Motions

Since buildings are typically instrumented at a limited number of floors, the motions of non-instrumented floors are computed by the cubic spline interpolation procedure (De la Llera and Chopra, 1998; Naeim, 1997). The cubic spline interpolation is performed on the building deformation relative to the base. Once the time variation of deformations of all floors have been developed using the cubic spline interpolation procedure, inter-story drifts at each time instant is computed from

$$\delta_j(t) = u_j(t) - u_{j-1}(t) \tag{1}$$

in which $\delta_j(t)$ is the inter-story drift in the $j^{th}$ story, and $u_j(t)$ and $u_{j-1}(t)$ are the deformations at the $j^{th}$ and $j$-1$^{th}$ floor levels at time *t*. The peak values of the drift in the $j^{th}$ story, $\delta_{jo}$, is computed as the absolute maximum value over time. These values, denoted as “derived” inter-story drifts, would be used to evaluate the NSP procedures.

For implementing the MPA procedure, contributions of various modes of the building to the total displacement are required. These contributions are extracted from the recorded (or interpolated) motions by using the standard modal analysis method (Chopra, 2001) as:

$$u_{jn}(t) = \frac{\boldsymbol{\phi}_n^T \boldsymbol{m}\boldsymbol{u}(t)}{\boldsymbol{\phi}_n^T \boldsymbol{m}\boldsymbol{\phi}_n} \phi_{jn} \tag{2}$$

in which $\boldsymbol{\phi}_n$ is the $n^{th}$ mode shape of the elastic building, $\boldsymbol{m}$ is the mass matrix, $\boldsymbol{u}$(t) is the vector of displacements at all floor levels at time *t*, and $\phi_{jn}$ is the n$^{th}$ mode shape component at the j$^{th}$ floor level. Once the contribution of the $n^{th}$ mode to the floor displacements have been computed, its contribution to inter-story drift, $\delta_{jn}(t)$, can be computed using Eq. (1). Further details of the procedure to analyze the recorded motions are available elsewhere (Goel, 2003a; 2003b; 2003c).

## Analytical Models

The computer program DRAIN-2DX (Prakash et al., 1993) was used for analysis of the selected buildings. A two dimensional model was developed for each of the selected buildings and calibrated by comparing the fundamental vibration periods obtained from the eigen-value analysis and from system-identification analysis of the initial "elastic" phase of the recorded motions. The accuracy of the model was further evaluated by comparing the time history of the floor displacements of the analytical model computed for the base acceleration and the time history floor displacements recorded during the earthquake. Details of the calibration process are available elsewhere (Goel, 2003c).

## Nonlinear Static Procedures

### FEMA-356 NSP

The nonlinear static procedure (NSP) specified in the FEMA-356 (ASCE, 2000) document may be used for any structure and any rehabilitation objective except for structures with significant higher mode effects. To determine if higher mode effects are present, two linear response spectrum analyses must be performed: (1) using sufficient modes to capture 90% of the total mass, and (2) using only the fundamental mode. If shear in any story from the first analysis exceeds 130% of the corresponding shear from the second analysis, the higher mode effects are deemed significant. In case the higher mode effects are present, the NSP analysis needs to be supplemented by the Linear Dynamic Procedure (LDP); acceptance criteria for the LDP are relaxed but remain unchanged for the NSP.

The FEMA-356 NSP requires development of a pushover curve, which is defined as the relationship between the base shear and lateral displacement of a control node, ranging between zero and 150% of the target displacement. The control node is located at the center of mass at the roof of a building. For buildings with a penthouse, the floor of the penthouse (not its roof) is regarded as the level of the control node. Gravity loads are applied prior to the lateral load analysis required to develop the pushover curve.

The pushover curve is developed for at least two vertical distributions of lateral loads. The first pattern is selected from one of the following: (1) Equivalent lateral force (ELF) distribution: $s_j^* = m_j h_j^k$ (the floor number $j = 1,2,\ldots$N) where $s_j^*$ is the lateral force and $m_j$ the mass at *j*th floor, $h_j$ is the height of the *j*th floor above the base, and the exponent $k = 1$ for fundament period $T_1 \le 0.5$ sec, $k = 2$ for $T_1 \ge 2.5$ sec; and varies linearly in between; (2) Fundamental mode distribution: $s_j^* = m_j \phi_{j1}$ where $\phi_{j1}$ is the fundamental mode shape component at the *j*th floor; and (3) SRSS distribution: $\mathbf{s}^*$ is defined by the lateral forces back-calculated from the story shears determined by linear response spectrum analysis of the structure including sufficient number of modes to capture 90% of the total mass. The second pattern is selected from either "Uniform" distribution: $s_j^* = m_j$ in which $m_j$ is the mass and $s_j^*$ is the lateral force at *j*th floor; or Adaptive distribution that changes as the structure is displaced. This distribution should be modified from the original distribution by considering properties of the yielded structure.

The target displacement is computed from

$$\delta_t = C_0 C_1 C_2 C_3 S_a \frac{T_e^2}{2\pi^2} g \tag{3}$$

where $T_e$ = Effective fundamental period of the building in the direction under consideration, $S_a$ = Response spectrum acceleration at the effective fundamental vibration period and damping ratio of the building under consideration and $g$ is the acceleration due to gravity, $C_0$ = Modification factor that relates the elastic response of an SDF system to the elastic displacement of the MDF building at the control node, $C_1$ = Modification factor that relates the maximum inelastic and elastic displacement of the SDF system, $C_2$ = Modification factor to represent the effects of pinched hysteretic shape, stiffness degradation, and strength deterioration, and $C_3$ = Modification factor to represent increased displacement due to P-delta effects.

The deformation/force demands in each structural element is computed at the target displacement and compared against acceptability criteria set forth in the FEMA-356 document. These criteria depend on the material (e.g., concrete, steel etc.), type of member (e.g., beam, column, panel zones, connections etc.), importance of the member (e.g., primary, or secondary) and the structural performance levels (e.g., immediate occupancy, life safety, collapse prevention).

The FEMA-356 NSP procedure contains several approximations. These include those in estimating the target displacement from Eq. 3, and using the pushover curve to estimate the member demands imposed by the earthquake. In this investigation, the focus is primarily on the second source of approximation. Therefore, the target displacement is selected to be equal to that of the roof level recorded during the earthquake, as opposed to calculating it according to the FEMA-356 document (Eq. 3). The structure is pushed to this target displacement using the FEMA-356 lateral load patterns and inter-story drifts are computed. The computed inter-story drifts are then compared with the "derived" inter-story drifts, i.e., those computed directly from the recorded motions using the procedure described in the preceding section. Such a comparison enables evaluation of the adequacy of various lateral load patterns in the FEMA-356 NSP, in particular, if the FEMA-356 NSP is able to capture the higher mode effects, which are likely to be present in the selected buildings.

**MPA Procedure**

Recently a MPA procedure has been developed to account for the higher mode effects and analytically tested for SAC buildings and ground motions (Chopra and Goel, 2002; Goel and Chopra, 2004). Following is a summary of this procedure.

1. Compute the natural frequencies, $\omega_n$ and modes, $\phi_n$, for linearly elastic vibration of the building.

2. For the *n*th-mode, develop the base shear-roof displacement, $V_{bn} - u_{rn}$, pushover curve for force distribution, $\mathbf{s}_n^* = \mathbf{m}\boldsymbol{\phi}_n$, where $\mathbf{m}$ is the mass matrix of the structure. Gravity loads, including those present on the interior (gravity) frames, are applied before the modal pushover analysis. The resulting P-Δ effects may lead to negative post-yielding stiffness in the pushover curve. Note the value of the lateral roof displacement due to gravity loads, $u_{rg}$.
3. Idealize the pushover curve as a bilinear curve. If the pushover curve exhibits negative post-yielding stiffness, the second stiffness (or post-yield stiffness) of the bilinear curve would be negative.
4. Convert the idealized $V_{bn} - u_{rn}$ pushover curve to the force-displacement, $F_{sn}/L_n - D_n$, relation for the *n*th -"mode" inelastic SDF system by utilizing $F_{sny}/L_n = V_{bny}/M_n^*$ and $D_{ny} = u_{rny}/\Gamma_n\phi_{rn}$ in which $M_n^*$ is the effective modal mass, $\phi_{rn}$ is the value of $\boldsymbol{\phi}_n$ at the roof, and $\Gamma_n = \boldsymbol{\phi}_n^T \mathbf{m}\mathbf{1} / \boldsymbol{\phi}_n^T \mathbf{m}\boldsymbol{\phi}_n$.
5. Compute the peak deformation $D_n$ of the nth-"mode" inelastic single-degree-of-freedom (SDF) system defined by the force-deformation relation developed in Step 4 and damping ratio $\zeta_n$. The elastic vibration period of the system is $T_n = 2\pi\left(L_n D_{ny}/F_{sny}\right)^{1/2}$. For an SDF system with known $T_n$ and $\zeta_n$, $D_n$ can be computed either by nonlinear RHA, from inelastic design spectrum, or by empirical equations for the ratio of deformations of inelastic and elastic systems (Chopra and Chintanapakdee, 2003).
6. Calculate peak roof displacement $u_{rn}$ associated with the *n*th-"mode" inelastic SDF system from $u_{rn} = \Gamma_n \phi_{rn} D_n$.
7. From the pushover database (Step 2), extract values of desired responses $r_{n+g}$ due to the combined effects of gravity and lateral loads at roof displacement equal to $u_{rn} + u_{rg}$.
8. Repeat Steps 3-7 for as many modes as required for sufficient accuracy.
9. Compute the dynamic response due to *n*th-"mode": $r_n = r_{n+g} - r_g$, where $r_g$ is the contribution of gravity loads alone.
10. Determine the total response (demand) by combining gravity response and the peak "modal" responses using the SRSS rule: $r \approx \max\left[ r_g \pm \left( \sum_n r_n^2 \right)^{1/2} \right]$.

Steps 3 to 6 of the MPA procedure described above are used to compute the peak roof displacement associated with the *n*th-"mode" inelastic SDF system. However, these steps are not necessary for analysis of a building for which recorded motions are available. The contribution of the *n*th-"mode" to the total roof displacement, $u_{rn}$, can be computed from modal decomposition of recorded motion using Eq. (2).

## Sum-Difference Procedure

The Sum-Difference procedure requires development of the pushover curve for force distribution given by

$$\boldsymbol{s} = \boldsymbol{s}_n \pm \boldsymbol{s}_r \tag{4}$$

in which $\mathbf{s}_n = \Gamma_n \boldsymbol{m}\boldsymbol{\phi}_n A_n$ with $\phi_n$ = $n$th mode shape, $\boldsymbol{m}$ = mass matrix, $\Gamma_n = \boldsymbol{\phi}_n^T \boldsymbol{m}\boldsymbol{1} / \boldsymbol{\phi}_n^T \boldsymbol{m}\boldsymbol{\phi}_n$, and $A_n$ = pseudo-acceleration of a linear elastic SDF system with period and damping ratio equal to that of corresponding to the $n$th mode of the building. Typically, values of $n$ = 1 and $r$ = 2 are used (Kunnath and Gupta, 2000). The floor displacements and story drifts are computed in a manner similar to that in the FEMA-356 NSP but utilizing the pushover curves for force distributions of Eq. (4).

## FEMA-356 Check for higher modes

The FEMA-356 criterion for checking presence of significant higher mode effects is applied to the four selected buildings. For this purpose, story shears are computed from two elastic modal analyses: (1) considering sufficient number of modes to capture at least 90% of the total mass, and (2) considering the fundamental mode only. For the Van Nuys building three modes were sufficient to capture 90% of the total mass, whereas five modes were needed for the Woodland Hills, Sherman Oaks, and Los Angeles buildings. The ratio of the story shears from the two analyses is computed and compared with the limiting value of 1.3 specified in the FEMA-356 document in Fig. 1 for the selected buildings. These results lead to the following conclusions.

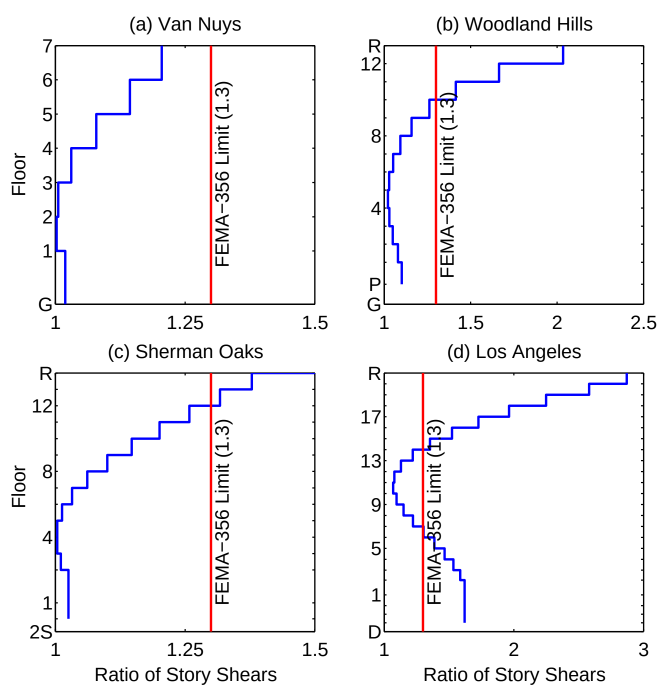


**Figure 1. Application of the FEMA-356 criterion to check the presence of higher mode effects in the selected buildings.**

The ratio of story shears from 3-mode analysis and 1-mode analysis is less than the FEMA-356 limiting value of 1.3 throughout the height of the Van Nuys building (Fig. 1a). Therefore, the FEMA-356 criterion indicates that higher mode effects should not be significant for this building. However, the ratio of story shears from 5-mode analysis and 1-mode analysis exceeds the FEMA-356 limiting value of 1.3 in upper stories of the Woodland Hills, Sherman Oaks, and Los Angeles buildings (Figs. 1b, 1c, and 1d); for the Los Angeles building this ratio exceeds the limiting value of 1.3 for the lower stories as well (Fig. 1d). Clearly, these buildings are expected to respond significantly in higher modes. Among these three buildings, the FEMA-356 criterion is barely exceeded in upper two stories of the Sherman Oaks building (Fig. 1c).

The results of Fig. 1 indicate that the FEMA-356 NSP is expected to provide sufficiently accurate estimates of the seismic demands for the Van Nuys building and perhaps for the Sherman Oaks building; the FEMA-356 higher mode criterion is satisfied throughout the height of the first building (Fig. 1a) and barely exceeded in upper two stories of the second building (Fig. 1c). However, the FEMA-356 NSP is not expected to give accurate seismic demands for the Woodland Hills and Los Angeles buildings because this criterion is significantly exceeded for these buildings (Figs. 1b and 1d). Since the FEMA-356 NSP is permitted for these buildings in conjunction with the LDP, the results from the FEMA-356 NSP are also included for these two buildings in this investigation.

## Pushover Curves

The lateral force distributions corresponding to four FEMA-356 NSP, two distributions in the Sum-Difference procedure ($\mathbf{s}_1 + \mathbf{s}_2$ and $\mathbf{s}_1 - \mathbf{s}_2$), and first three modes of the MPA procedure are used to generate pushover curves for the longitudinal frame on the south face of the Van Nuys building, the frame in the north-south direction of the Woodland Hills building, the longitudinal frame in the east-west direction of the Sherman Oaks building, and the braced frames in the north-south direction of the Los Angeles buildings. The first initiation of yielding in beams, columns, connections, or brace (buckling in compression) is also indicated on each pushover curve. The pushover curves presented in Figs. 2 to 4 lead to the following observations.

The characteristic – elastic stiffness, yield strength, and yield displacement – of the pushover curve depend on the lateral force distribution (Fig. 2). The “Uniform” distribution generally leads to pushover curve with higher elastic stiffness, higher yield strength, and lower yield displacement compared to all other distributions. The ELF distribution, on the other hand, leads to pushover curve with lower elastic stiffness, lower yield strength, and higher yield displacement. The “Mode” 1 and SRSS distribution give pushover curves that are bounded by the pushover curves due to “Uniform” and ELF distributions.

For the Van Nuys and Sherman Oaks buildings (Figs. 2a and 2c), the “Mode” 1 and SRSS pushover curves are essentially identical. For the Woodland Hills building (Fig. 2b), the two curves are essentially identical up to the elastic limit. Thereafter, the strength is higher for the SRSS distribution compared to the “Mode” 1 distribution. For the Los Angeles building (Fig. 2d), the “Mode” 1 curve is essentially identical to the ELF curve.

The pushover curves for the Woodland Hills and Sherman Oaks buildings (Figs. 2b and 2c) exhibit significant degradation in lateral load carrying capacity at large roof displacements.

The onset of the degradation depends on the lateral force distribution: the "Uniform" distribution induces the earliest, the ELF distribution the latest, and the "Mode" 1 and SRSS distributions in between the "Uniform" and ELF distributions. The degradation in the lateral load carrying capacity occurs due to P-Delta effects arising from the gravity loads. These effects may lead to negative slope of the pushover curve at large roof displacements, as apparent for the Woodland Hills and Sherman Oaks buildings (Figs. 2b and 2c).

In the Van Nuys building, the first yielding is initiated in the beams; the first yielding of columns occurs at much larger displacements (Fig. 2a). The first yielding in the Woodland Hills building occurs in the connection followed soon after by the first yielding of the beam (Fig. 2b). The columns start to yield at much higher deformation level, followed immediately by rapid deterioration of the lateral load carrying capacity of the building. The first yielding in the Sherman Oaks building occurs in the beam followed soon after by the first yielding of the column (Fig. 2c). The yielding in the Los Angeles building initiates at very low deformation levels due to buckling of the compression braces (Fig. 2d). The columns yield at much higher deformation level.

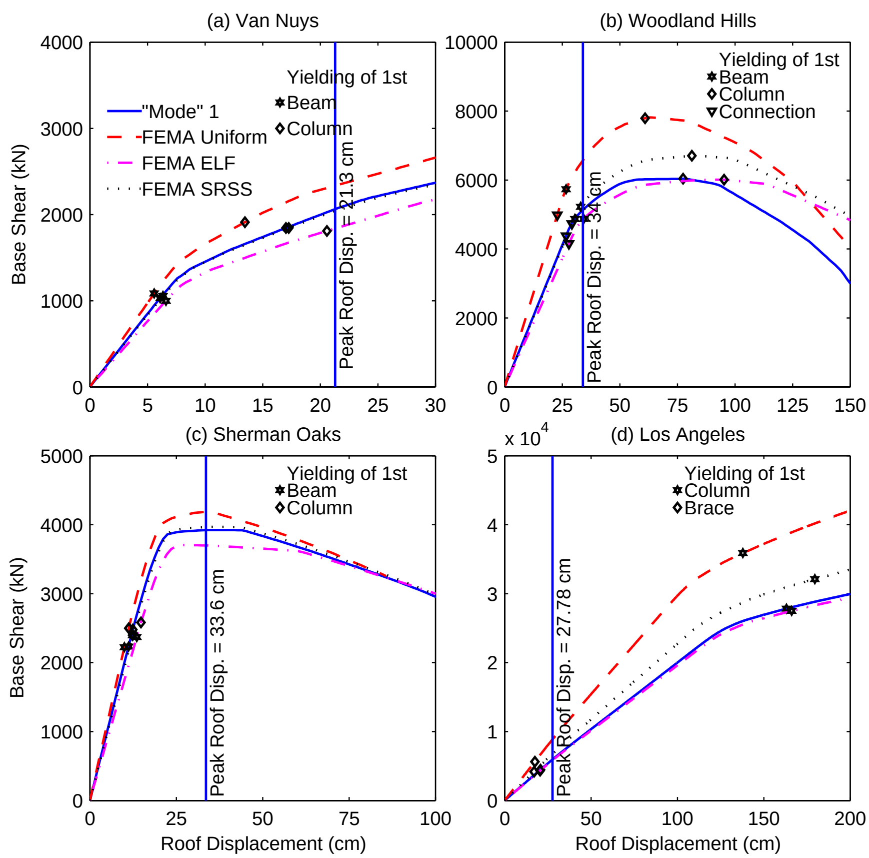


**Figure 2. Pushover curves for the four FEMA-356 distributions.**

The peak displacement recorded at the roof of each selected building during the 1994 Northridge earthquake is also shown in Fig. 2. These results indicate that the Van Nuys and the Sherman Oaks buildings are deformed significantly beyond the elastic limit during the 1994 Northridge earthquake, as apparent from the peak roof displacement being much larger than the yield displacement (Figs. 2a and 2c). The Woodland Hills building is deformed only slightly beyond the elastic limit (Fig. 2b), and the Los Angeles building responded essentially in the elastic range (Fig. 2d), except for buckling of few braces, during the 1994 Northridge earthquake.

The pushover curves for the two Sum-Difference distributions (Fig. 3) exhibit significantly different characteristics. The pushover curve for the mode 1+2 distribution exhibits significantly larger initial stiffness and much higher yield strength compared to the mode 1–2 distribution. These differences are much larger than those noted previously (Fig. 2) for four different FEMA-356 distributions. While the mode 1+2 distribution led to significant strength degradation at large roof displacements in the Woodland Hills and the Sherman Oaks buildings (Figs. 3b and 3c) – a pattern similar to that noted for these two buildings for the FEMA-356 distributions (Figs. 2b and 2c) – the mode 1–2 distribution leads to very little degradation in strength. Furthermore, the first yielding in columns is initiated at significantly larger roof displacements due to the mode 1–2 distribution in three of the four buildings (Figs. 3a, 3b, and 3d).

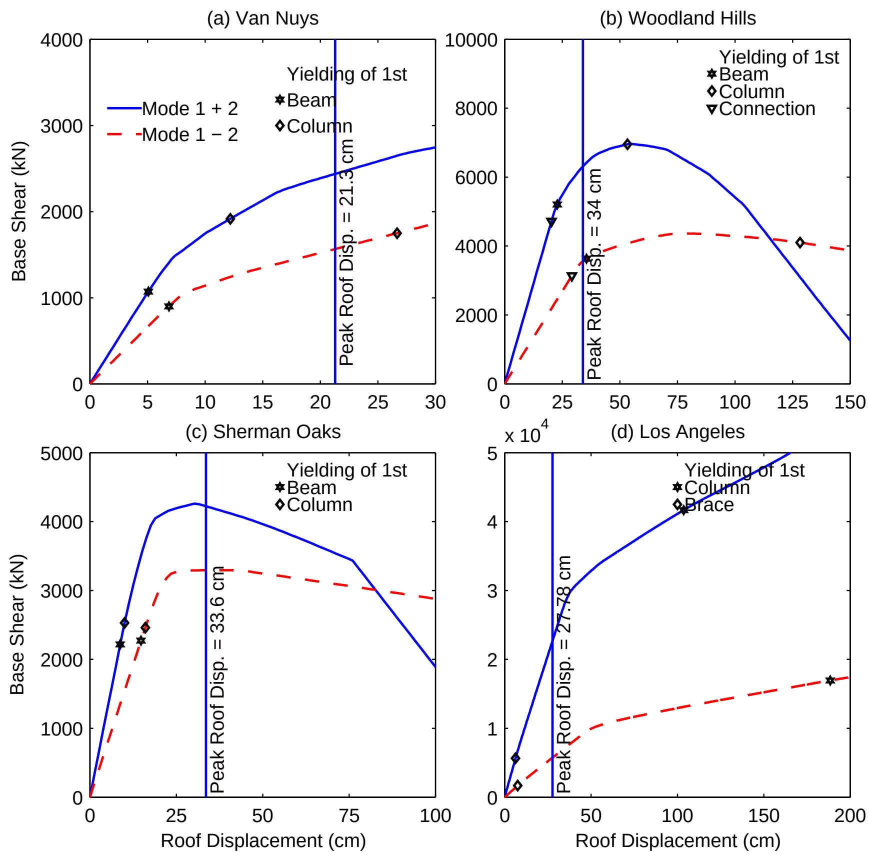


**Figure 3. Pushover curves for the two distributions in the Sum-Difference procedure.**

The "modal" pushover curves are shown in Fig. 4. Included on each pushover curve is the peak value of the modal component of the roof displacement recorded during the 1994 Northridge earthquake; the modal component is computed from Eq. (2). For example, the peak values of the first, second, and third mode contributions to the total roof displacements were 21.1 cm, 2.93 cm, and 2.75 cm, respectively, during the 1994 Northridge earthquake for the Van Nuys building. These values are shown on pushover curves for each of the three modes of the Van Nuys building (Fig. 4a).

The "modal" pushover curves show that the Van Nuys building (Fig. 4a) experienced significant yielding in the first "mode". The building is deformed nearly to the elastic limit of the pushover curve in the second and third modes. However, yielding in these modes has been initiated in some beams and columns, indicating that modes higher than the fundamental mode also contributed to the inelastic behavior of this building. While the Woodland Hills, and Sherman Oaks buildings are deformed beyond the elastic limit only in the first mode (Figs. 4b, and 4c), these buildings remain elastic in the higher modes with the roof displacement component during the 1994 Northridge earthquake being smaller than that required to induce yielding in any element. The Los Angeles building remains essentially elastic in all modes (Fig. 4d). However, the peak roof deformation during the 1994 Northridge earthquake was found to be slightly larger than that required for first bucking in the compression braces for all modes.

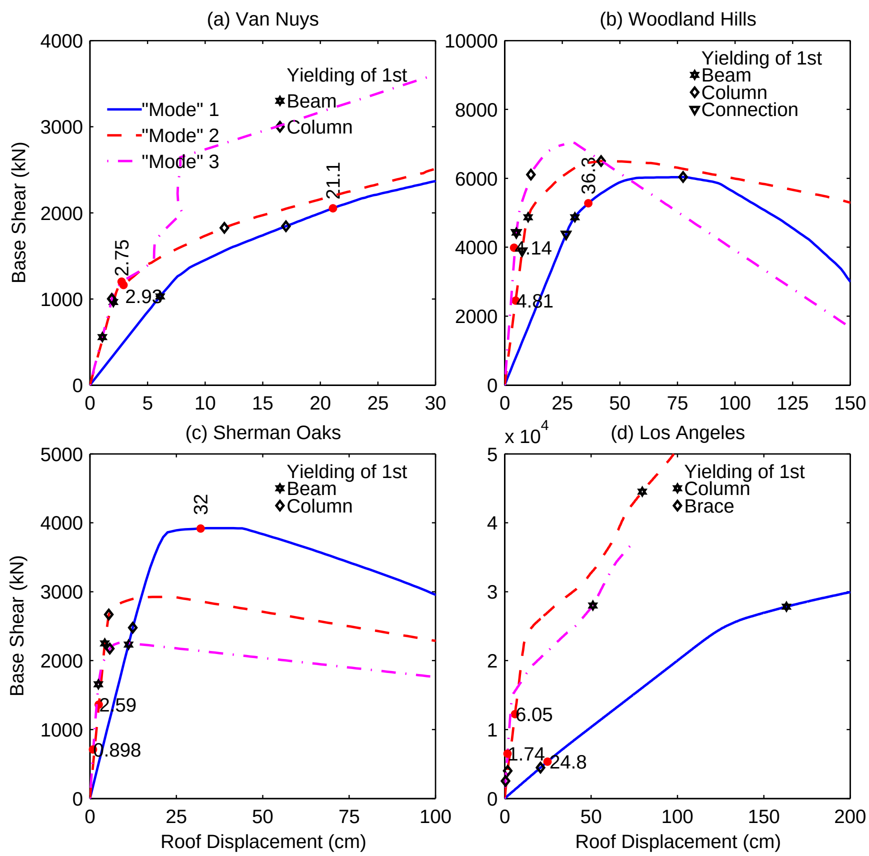


**Figure 4. Pushover curves for the three modal distributions in the MPA procedure.**

As noted previously, none of the selected buildings responded beyond the elastic limit in modes higher than the fundamental mode. For such buildings, the Modified Modal Pushover Analysis (MMPA), wherein the response contributions of the modes higher than the fundamental mode are computed by assuming the building to be linearly elastic, may be used to estimate the seismic demands (Chopra et al., 2004). The MMPA procedure is an attractive alternative to the

MPA procedure for these buildings because of reduced computational efforts; the pushover curves for higher modes are not needed in the MMPA procedure.

The pushover results presented so far also show that while Van Nuys, Woodland Hills, and Sherman Oaks buildings were deformed beyond the elastic limit during the 1994 Northridge earthquake, the Los Angeles building remained essentially elastic during this earthquake. Therefore, the NSP procedures – developed for estimating seismic demands in buildings deformed beyond the elastic limits – may not be strictly applicable for the Los Angeles building. However, the MPA procedure becomes equivalent to the standard Response Spectrum Analysis (RSA) procedure for buildings responding in the linear elastic range (Chopra and Goel, 2001). Therefore, the Los Angeles building provides important data for evaluating applicability of the MPA procedure for linear elastic buildings and is included in this investigation. Although results from the FEMA-356 NSP and the Sum-Difference procedure may not be strictly valid for this building, they are included nonetheless for comparison purposes.

## Evaluation of Nonlinear Static Procedures

The nonlinear static procedures are evaluated by comparing the story drifts from the four FEMA-356 analyses, two Sum-Difference analyses, and the MPA procedure with the "derived" values from the recorded motions. The target roof displacement in the FEMA-356 and the Sum-Difference analyses was selected to be that "derived" from the motions recorded at the roof. Furthermore, *n*th-"mode" component of the roof displacement, $u_{rn}$, required in the MPA procedure was taken to be the value obtained from the *n*th "modal" decomposition of the recorded motions. It is useful to emphasize that since two-dimensional models of the buildings have been used in this investigation, the computed and recorded motions examined are those at the center of the each building. Although the FEMA-356 criterion for higher mode effects is significantly exceeded for at least two of the four selected buildings, results from the FEMA-356 NSP are included because such analyses are permitted in conjunction with the LDP analysis.

The comparison of story drifts from the FEMA-356 analyses and the recorded motions (Fig. 5) show that the FEMA-356 force distributions typically lead to gross underestimation of drifts in the upper stories of all of the four selected buildings. Among the four FEMA-356 distributions, the "Uniform" force distribution almost always leads to the worst estimates of story drifts (Fig. 5). This distribution leads to underestimation of the drift at 7$^{th}$ story of the Van Nuys building by more 90% – the story drifts from recorded motions and FEMA-356 "Uniform" distributions are 4.11 cm and 0.32 cm, respectively (Fig. 5a); underestimation of the drift in the top story of the Woodland Hills building by about 67% – the story drifts from recorded motions and FEMA-356 "Uniform" distributions are 3.01 cm and 1.02 cm, respectively (Fig. 5b); underestimation of the drift in the top story of the Sherman Oaks building by more than 80% – the story drifts from recorded motions and FEMA-356 "Uniform" distributions are 1.51 cm and 0.24 cm, respectively (Fig. 5c); and underestimation of the drift in the top story of the Los Angeles buildings by more than 40% – the story drifts from recorded motions and FEMA-356 "Uniform" distributions are 2.86 cm and 1.55 cm, respectively (Fig. 5d). Therefore, usefulness of the "Uniform" distribution in the FEMA-36 NSP should be re-examined. A similar observation was also made in an earlier study based on analytical response of six buildings with steel moment-resisting frames (Goel and Chopra, 2004).

The FEMA-356 NSP also leads to significant overestimation of the drift in lower stories of the Van Nuys and Sherman Oaks building (Figs. 5a and 5c) with the "Uniform" distribution leading to the largest overestimation. For example, the "Uniform" distribution leads to overestimation of the drift in the first story of the Van Nuys building by about 50% – the story drifts from recorded motions and FEMA-356 "Uniform" distributions are 4.80 cm and 7.23 cm, respectively (Fig. 5a); and overestimation of the drift in the first story of the Sherman Oaks building by more than 50% – the story drifts from recorded motions and FEMA-356 "Uniform" distributions are 8.05 cm and 13.60 cm, respectively (Fig 5c).

The presented results for story drifts of the Van Nuys building (Fig. 5a) also demonstrate another serious limitation of the FEMA-356 NSP. The higher mode effects for this building were deemed not to be significant based on the FEMA-356 criterion (Fig. 1a). Therefore, expectation was that the FEMA-356 would lead to reasonable estimates of drifts throughout the building height. Yet the drifts are significantly underestimated in upper stories by the FEMA-356 NSP (Fig. 5a). Since the larger drifts in upper stories tend to occur due to higher modes, it appears that higher mode effects were significant for this building and the FEMA-356 criterion apparently failed to identify these effects. This indicates that the FEMA-356 criterion for significant higher mode effects should be re-examined.

The inability of the FEMA-356 NSP in accurately estimating the drifts in upper stories of the Woodland Hills and Los Angeles buildings – the two buildings for which the FEMA-356 criterion for higher modes is significantly exceeded (Figs. 1b and 1d) – validates the well-known limitation that the FEMA-356 NSP is not applicable for buildings with significant higher mode effects. The authors of FEMA-356 clearly acknowledged this limitation of the FEMA-356 NSP procedure and required that the results of the NSP analyses be supplemented by the results of the LDP analysis for such buildings.

The story drifts from the two Sum-Difference analyses presented in Fig. 6 show that the mode 1+2 distribution gives larger drifts in lower stories and the mode 1–2 distribution leads to larger drifts in upper stories. This observation for the four selected buildings is consistent with that based on one building in an earlier investigation (Kunnath and Gupta, 2000).

It is expected that the envelope of the responses from the two analyses in the Sum-Difference procedure will provide reasonable estimates of the seismic demands throughout the building height. However, comparison of the story drifts from the two Sum-Difference analyses with those from the recorded motions (Fig. 6) shows that this may not always be the case. The Sum-Difference method generally leads to underestimation of the drift in upper few stories of all selected buildings (Fig. 6) with the underestimation being slightly smaller compared to the FEMA-356 NSP analyses (Fig. 5); for the Los Angeles building, however, the Sum-Difference procedures may overestimate the drifts in some upper stories (Fig. 6d). Furthermore, the Sum-Difference method almost always significantly overestimates the drifts in lower stories of all selected buildings (Fig. 6) with the overestimation being much larger than that from the FEMA-356 analyses (Fig. 5).

The results presented for the Sum-Difference procedure indicate that although this procedure tends to give improved drifts in upper stories compared to the FEMA-356 NSP, this procedure is still not accurately able to capture the higher mode effects. Furthermore, the Sum-

Difference procedure provides overestimation of drifts in lower stories of the selected buildings that is worse than that from the FEMA-356 NSP.

The MPA procedure for three of the four selected buildings – Van Nuys, Woodland Hills, and Los Angeles – provides estimates of drifts in most stories that are better than those from the FEMA-356 NSP (Figs. 5a, 5b, and 5c) and the Sum-Difference procedure (Figs. 6a, 6b and 6d). In particular, the match between the drifts from MPA and recorded motions is reasonable good in upper stories indicating that the MPA procedure is able to capture the higher mode effects for these buildings. However, significant discrepancy may exist in few stories, such as drift in the 6th story of the Van Nuys building (Fig. 5a), and top stories of Woodland Hills and Los Angeles buildings (Figs. 5b and 5d). The reasons behind this discrepancy are examined latter in this section.

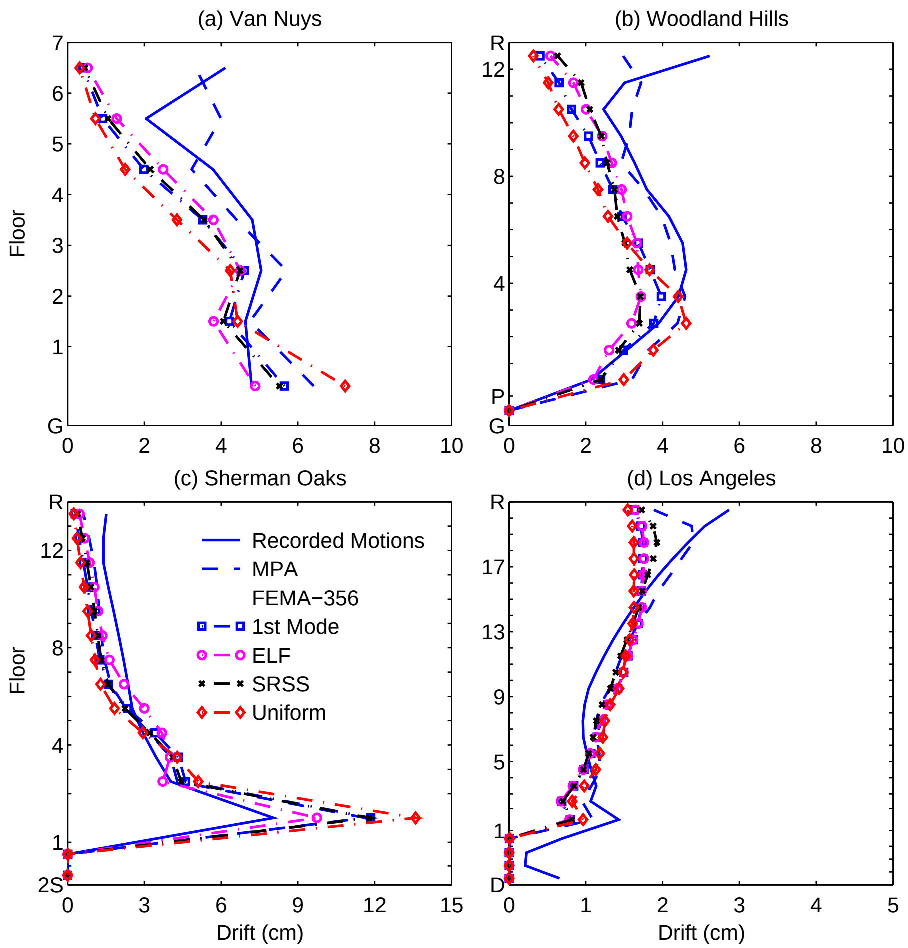


Figure 5. Comparison of story drifts from recorded motions, MPA procedure, and four FEMA-356 NSP for four distributions.

For the Sherman Oaks building, the MPA procedure provides estimates of the story drifts slightly better than those from the FEMA-356 NSP (Fig. 5c) or the Sum-Difference procedure (Fig. 6c). Although not apparent from Fig. 5c, the overestimation of drifts in lower stories and underestimation of drifts in upper stories from the MPA procedure is smaller compared to the FEMA-356 NSP. Furthermore, the overestimation in drifts in lower stories is much smaller from the MPA procedure compared to the Sum-Difference procedure (Fig. 6c). Yet the results from the MPA procedure are significantly different compared to those from the recorded motions for this building.

The results presented for story drifts of the Sherman Oaks building indicate that the behavior of this building is dominated by the effects of "soft" first story. A large concentration of drift occurs in the first story (Figs. 5c and 6c) both in results from recorded motions as well as NSP analyses; drifts is upper stories are only a small fraction of the drift in the first story. For such a building, where "soft" story effects dominate, all the nonlinear static procedure – the FEMA-356 NSP, the Sum-Difference, and the MPA – failed to provide reasonable estimate of story drifts: these procedures overestimate the drifts in the first story and underestimate them in the upper stories.

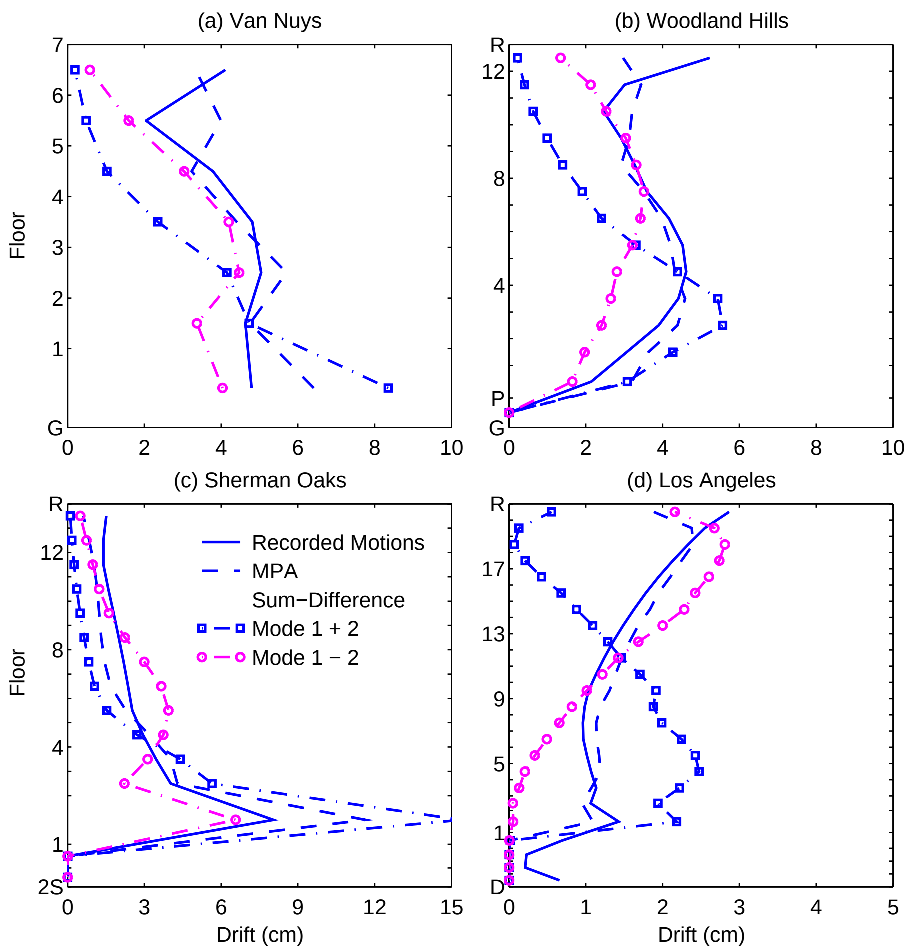


Figure 6. Comparison of story drifts from recorded motions, MPA procedure, and two distributions in the Sum-Difference procedure.

As noted previously, while the estimates of story drifts from the MPA procedure are much better compared to the FEMA-356 NSP, significant differences exist in a few stories. In order to understand the source of this discrepancy, peak drifts in each mode of the MPA procedure are compared with those obtained from modal decomposition of recorded motions (Fig. 17). This comparison shows that the match between the two is reasonably good. Therefore, the prime source of discrepancy appears to be from modal combination rule used in the MPA procedure.

A fraction of the errors in the modal combination may be attributed to application of the modal combination rule, which is strictly valid for elastic buildings, for buildings responding beyond the elastic range. However, this fraction has been found to be small in an earlier study where errors in the MPA results of elastic and inelastic systems were compared (Goel and Chopra, 2004).

The error in large part appears to be due to application of the modal combination rule for peak responses of a single ground motion. Note that the modal combination rules are based on random vibration theory and the combined peak response should be interpreted as the mean of the peak values of response to an ensemble of earthquake excitations. Thus, the modal combination rules are intended for use when the excitation is characterized by a smooth response (or design) spectrum. Although modal combination rules can also approximate the peak response to a single ground motion characterized by a jagged response spectrum, the errors are expected to be much larger in some cases, as noted in this investigation.

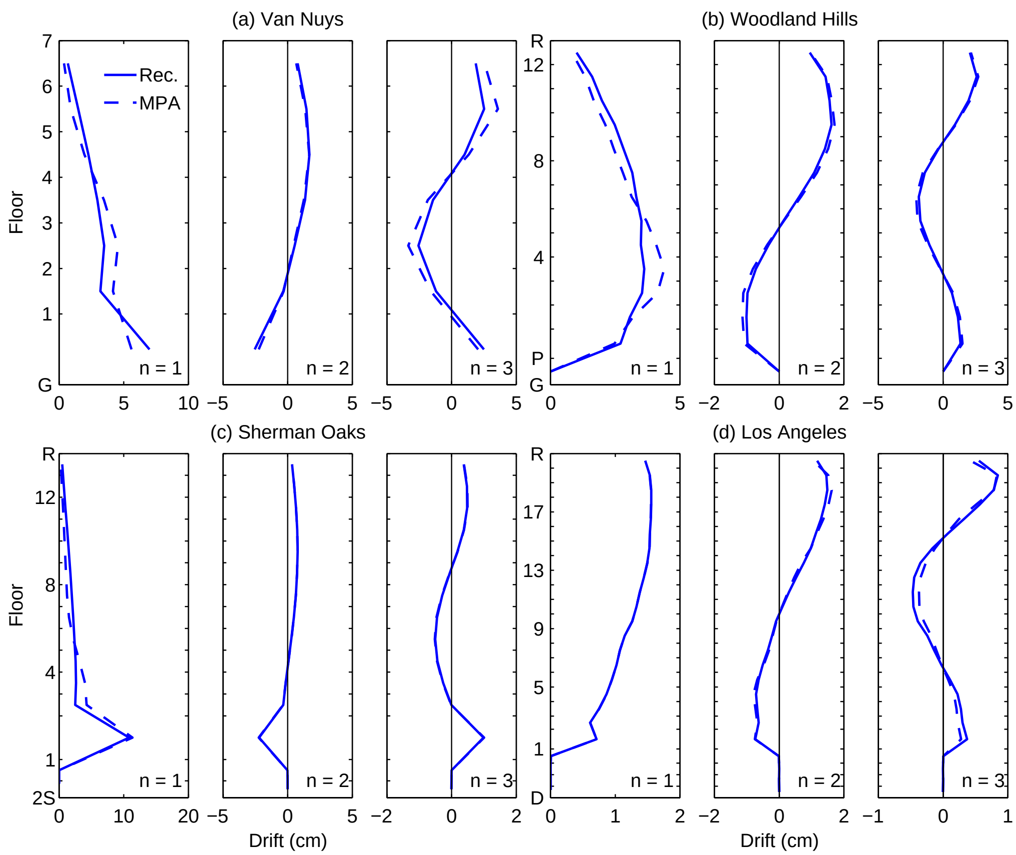


Figure 7. Comparison of story drifts from MPA procedure with results derived from modal decomposition of recorded motions for first three modes ($n$ = 1, 2, and 3).

It is useful to note that while the total drifts in first story of the Sherman Oaks building is significantly overestimated by the MPA procedure (Fig. 5c), the mode-by-mode match between the recorded motions and the MPA procedure is excellent even for this building (Fig. 7c).

Furthermore, each "modal" analysis in the MPA procedure is able to capture the "soft" story effects, as apparent from the concentration of drifts in first story of this building in results for each mode (Fig. 7c).

## Conclusions

This research investigation evaluated three nonlinear static procedures (NS) – the FEMA-356 NSP, the Sum-Difference, and the MPA – using recorded motions of four buildings that were damaged during the 1994 Northridge earthquake. The selected buildings were analyzed using the four distributions – "Uniform", ELF, SRSS, and 1$^{st}$ "Mode" – in the FEMA-356 NSP, two distributions – mode 1+2 and mode 1–2 – in the Sum-Difference procedure, and the MPA procedure. First the pushover curves were examined followed by comparison of story drifts from the three NSP with those from the recorded motions. This investigation has led to the following conclusions:

The pushover curves indicate that the elastic stiffness, yield strength and yield displacement depend on the lateral force distribution. Among the FEMA-356 distributions, the "Uniform" distribution generally leads to pushover curve with higher elastic stiffness, higher yield strength, and lower yield displacement compared to all other distributions; the ELF distribution leads to pushover curve with lower elastic stiffness, lower yield strength, and higher yield displacement; and the "Mode" 1 and SRSS distribution pushover curves are bounded by the pushover curves due to "Uniform" and ELF distributions. Among the Sum-Difference distributions, mode 1+2 distribution leads to significantly larger initial stiffness and much higher yield strength compared to the mode 1–2 distribution. Furthermore, the pushover curves for the Woodland Hills and Sherman Oaks buildings exhibit significant degradation in lateral load carrying capacity at larger roof displacements due to P-Delta effects arising from the gravity loads. Among the four FEMA-356 distributions, the "Uniform" distribution induces the earliest degradation in the lateral load carrying capacity. While the mode 1+2 distribution leads to significant strength degradation at large roof displacements, the mode 1–2 distribution induces very little degradation in strength.

Comparison of the elastic limits of various buildings with the peak roof displacements recorded during the 1994 Northridge earthquake indicates that the Van Nuys and the Sherman Oaks buildings are deformed significantly beyond the elastic limit, the Woodland Hills building is deformed only slightly beyond the elastic limit, and the Los Angeles building responded essentially in the elastic range, except for buckling of few braces. Furthermore, three of the four selected buildings – Van Nuys, Woodland Hills, and Sherman Oaks – are deformed beyond the elastic limit only in the first mode whereas the Los Angles building remained elastic in all modes during the 1994 Northridge earthquake.

The comparison of the story drifts from the NSP with those from the recorded motions showed that the FEMA-356 NSP led to gross underestimation of drifts in upper stories of all four selected building and significant overestimation of drifts in lower stories of the two of the four buildings. The underestimation in upper stories ranges by 90% for the Van Nuys building to about 40% for the Los Angeles building, and overestimation in lower stories by about 50% occurred for Van Nuys and Sherman Oaks buildings. Among the four FEMA-356 distributions, the "Uniform" force distribution leads to the most excessive underestimation in upper stories and overestimation in the lower stories. Therefore, usefulness of the "Uniform" distribution in the FEMA-36 NSP should be re-examined.

The presented results also confirm the well-known limitation that the FEMA-356 NSP is not applicable for buildings with significant higher mode effects. The authors of FEMA-356 clearly acknowledged this limitation of the FEMA-356 NSP procedure and required that the results of the NSP analyses be supplemented by the results of the LDP analysis for such buildings.

The FEMA-356 NSP is expected to provide reasonable estimate of the response if the effects of higher modes are deemed not to be significant based on the FEMA-356 criterion. Although the FEMA-356 criterion is clearly satisfied for the Van Nuys building and nearly satisfied for the Sherman Oaks building, the drifts in upper stories are still significantly underestimated indicating the need to re-examine the FEMA-356 criterion for evaluating significant higher mode effects.

The Sum-Difference procedure provides better estimates of drifts in upper stories compared to the FEMA-356 NSP. However, this procedure is still not accurately able to capture the higher mode effects. Furthermore, the Sum-Difference procedure provides overestimation of drifts in lower stories of the selected buildings that is worse than that from the FEMA-356 NSP.

The MPA procedure provides estimates of drifts that are better throughout the building height, with exceptions in a few stories, compared to those from the FEMA-356 NSP and the Sum-Difference procedure. Furthermore, the MPA procedure is able to account for the higher mode effects. This suggests that the limitation that the NSP be used only for buildings for which the response is controlled by the fundamental mode may be removed if the MPA procedure is used to compute the seismic demands.

Finally, all NSP procedures – FEMA-356 NSP, Sum-Difference, and the MPA – failed to provide accurate estimates of story drifts in the building with dominant "soft" first story effects. Therefore, the application of the NSP to such buildings should be carefully examined.

The response for each mode in the MPA procedure matched closely with the modal response obtained from decomposition of the recorded motions, indicating the observed discrepancy between the response from MPA and recorded response is due to limitations in the combination procedure. The modal combination rules are based on random vibration theory and the combined peak response should be interpreted as the mean of the peak values of response to an ensemble of earthquake excitations. Thus, the modal combination rules are intended for use when the excitation is characterized by a smooth response (or design) spectrum. Applied to the peak response to a single ground motion characterized by a jagged response spectrum, the errors are expected to be much larger in some cases, as noted in this investigation.

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