CRITICAL ASSESSMENT OF CODE TORSIONAL PROVISONS USING CSMIP DATABASE OF INSTRUMENTED BUILDINGS

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Abstract

This research aims at assessing the validity of accidental torsion provisions in building codes. Uncertainty in stiffness is considered as the main source of possible eccentricity. Monte Carlo simulation is utilized to statistically assess the behavior of nine one-story symmetric-in-plan base systems with nine different translational to rotational period ratios (Ω). Three vibrational characteristics and equivalent design eccentricity are developed and compared with information obtained from CSMIP database. The effect of Ω, plan aspect ratio, and correlation in building stiffness on building displacement amplification due to torsion are investigated. Equivalent design eccentricity is quantified based on these results.

Introduction

Seismic code provisions require that the effect of building torsion during seismic excitation be considered at each floor level. This effect for symmetric-in-plan buildings is captured by exerting the seismic equivalent lateral force of each floor at a distance–equal to 5% of the building's plan dimension perpendicular to the direction of the applied equivalent lateral force–from the center of mass (CM) of the floor diaphragm. Denoted as "accidental torsion," the later represents the effect of discrepancies between the mass and stiffness distribution along the height of the real building, and the effect of rotational component of ground shaking on the structure. In essence, accidental torsion is rooted in the inherent uncertainty of engineering models. Traditionally, analytical methods for seismic response assessment of buildings are incapable of addressing such modeling uncertainty, unless accidental torsion is explicitly built into the analytical model through the so-called 5% rule.

Building code provisions on inclusion of accidental torsion in seismic response assessment of buildings is not limited to symmetric-in-plan structures. Accidental torsion is added to the inherent torsional effects in asymmetric-in-plan buildings, and is explicitly considered in traditional analytical seismic response assessment methods. In contrast, however, the effect of modeling uncertainty that leads to consideration of accidental torsion is dwarfed by the inherent torsional effects in buildings with asymmetric-in-plan. Therefore, its critical to maintain focus on assessing the torsional response of symmetric-in-plan buildings.

This research is built on the results of previous work at CSMIP (De la Llera and Chopra, 1992), and others (e.g. De la Llera and Chopra, 1994, and 1995; Lin et al., 2001; Hernandez and Lopez, 2004; De-la-Colina and Almeida, 2004; Basu et al., 2014) that lend itself to evaluation of code-accidental torsion provisions and its dependence on structural system properties.

Specifically, in an effort similar to what is proposed here, De la Llera and Chopra (1992) concluded–based on motions recorded in three buildings instrumented by CSMIP–that code specified accidental torsion is adequate in representing the torsion in recorded motions. They speculated that it is not necessary to consider accidental torsion in the design of many buildings. We critically evaluate these claims using CSMIP data, and develop expressions for accidental torsion of buildings. The ultimate aim of this study is to develop a set of rational, meaningful, and practical ways to include accidental torsion in seismic response assessment, and improve both the seismic design provisions of building codes (e.g., ASCE, 2010, and 2007) and the practice of performance-based design and retrofit of structures.

Methodology

For a building subject to translational ground motions, the equation of motion can be written as Eq.1-1:

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \\ 0 & 0 & I_o \end{bmatrix} \begin{Bmatrix} \ddot{u}_x \\ \ddot{u}_y \\ \ddot{u}_\theta \end{Bmatrix} + \begin{bmatrix} k_{xx} & 0 & k_{x\theta} \\ 0 & k_{yy} & k_{y\theta} \\ k_{\theta x} & k_{\theta y} & k_{\theta\theta} \end{bmatrix} \begin{Bmatrix} u_x \\ u_y \\ u_\theta \end{Bmatrix} = - \begin{Bmatrix} m\ddot{u}_{gx}(t) \\ m\ddot{u}_{gy}(t) \\ 0 \end{Bmatrix} \quad \text{(Eq.1-1)}$$

where $I_o = mr^2$ is the rotational inertia. Theoretically, a building with symmetric plan has zero off-diagonal elements in the stiffness matrix, and the dynamic responses of two translational directions of the system are uncoupled. However, when asymmetric stiffness distribution takes place in a system with nominally symmetric plan, translational ground motion triggers torsional vibration and the deformation along the translational axis is amplified due to torsional effects.

One of the most important parameters that affects torsional behavior of a building is Ω, the ratio of dominant translational period to dominant rotational period. Large Ω values are associated with perimeter frame buildings with large torsional stiffness (Eq.1-2), while small Ω values represent buildings such as core wall systems with low torsional stiffness. Ω values ranging from 0.6 to 1.4 are investigated in this study, which covers most of the building cases.

$$\Omega = \frac{T_{tran}}{T_{rot}} = \frac{\sqrt{m/K_{tran}}}{\sqrt{mr^2/K_{rot}}} \quad \text{(Eq.1-2)}$$

Nine one-story four-VLLR (i.e. vertical lateral load resisting elements) base systems with symmetric plans are developed whose translational period T_{tran} = 1.5sec and Ω = 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2, 1.3, 1.4, respectively. The deformation of the roof of the base systems subject to one directional earthquake ground motion along the direction of T_{tran}, denoted as δ_b, is merely the translational displacement along that direction due to the symmetric plan of the buildings.

To consider the effect of uncertainty in stiffness of VLLRs, literature review has been conducted to access the variability of element cross section dimensions, second moment of inertia and material strength. Ramsay *et al.* (1979) used Monte Carlo simulation to conclude that deformation of reinforced concrete beams has a coefficient of variation of 0.14, and De la Llera and Chopra (1994) suggested the same value be used as the variation of stiffness of reinforced concrete elements. This approach is under the assumption that force distribution is deterministic (a conservative estimate). Ellingwood and Galambos (1980) investigated probability based load criterion and Melchers (1987) evaluated reliability of structures, showing that the coefficient of variation of Young's modulus and cross section moment of inertia is approximately 0.06 and 0.05, respectively. Bournonville *et al.* (2004) performed statistical analysis to mechanical properties of reinforcing bars and found that coefficient of variation of reinforcement yield strength ranges from 0.03 to 0.09. ASTM A6 (2005) provides variability of structural element dimensions, and ASTM A992 (2004) provides steel and concrete material strength; according to the ASTM resources, section depth or width has a coefficient of variation ranging from 0.01 to 0.04, and column steel yield stress (Grade 50) has a coefficient of variation of 0.05.

If each dimension is assumed to have a coefficient of variation equal to 0.03, then moment of inertia has a coefficient of variation of 0.06 to 0.09 assuming no correlation and complete correlation between each dimension, respectively. Given the coefficient of variation of Young's modulus is approximately 0.06, coefficient of variation of the stiffness of all structural elements are conservatively equal to 0.14.

A hundred sample Monte Carlo simulations is run for each base system, treating four VLLRs as four normally distributed random variables, with mean values equal to the stiffness of the base system and the coefficient of variation equal to 0.14. Given that the probability of the stiffness of one of the four VLLRs being close to the stiffness of another is high if they are manufactured in batch and produced identically, a correlation coefficient of $\rho = 0.5$ between the stiffness of four VLLRs is assumed. For comparison purposes, $\rho = 0$, which conservatively assumes uncorrelated VLLR stiffness are also used in this study. Forty ground motion are scaled to a low intensity level of S_a=0.06g at a period of 1.5sec to examine linear torsional behavior of the buildings. Therefore, 4000 asymmetric building plan analyses are performed per base system and 36000 analyses are completed in total. These 36000 analyses are repeated for base systems with plan aspect ratios of 1:1, 1:2, 1:4 and 1:8 to account for the effect of building plan dimension, where aspect ratio is defined as the length along the applied ground motion direction to the length perpendicular to the ground motion direction.

Effect of torsional vibration is measured and estimated via the largest amplification in displacement among four corners of the building at the roof level. With asymmetric stiffness distributions, buildings rotate even though the ground motion is applied to the center of mass. Total response is the summation of translational response and rotational response. Three torsional vibration characteristics are developed in Eq.1-3:

$$\alpha_1^i = \frac{max(\delta_{tran}^i + \delta_{rot}^i)}{max(\delta_b)} \quad \text{(Eq.1-3,a)}$$

$$\alpha_2^i = \frac{max(\delta_{tran}^i + \delta_{rot}^i)}{max(\delta_{tran}^i)} \quad \text{(Eq.1-3,b)}$$

$$\alpha_3^i = \frac{max(\delta_{tran}^i)}{max(\delta_b)} \quad \text{(Eq.1-3,c)}$$

$$\alpha_1^i = \alpha_2^i \alpha_3^i \quad \text{(Eq.1-3,d)}$$

α_1^i is the ratio of the peak total response of the i^{th} asymmetric system to the peak translational response of the base system without eccentricity. α_2^i is the ratio of the peak total response of the i^{th} asymmetric system to the peak translational response of the same i^{th} system. α_3^i is the ratio of the peak translational response of the i^{th} asymmetric system to the peak translational response of the base system. To summarize, α_1^i estimates the total displacement amplification due to stiffness eccentricity compared to a non-eccentric base system; it is the multiplication of α_2^i and α_3^i, where the former estimates the displacement amplification within a certain asymmetric system, and the latter estimates the contribution of pure translational displacement in that asymmetric system.

Torsional vibration characteristics represent the amplification in displacement due to stiffness uncertainty in asymmetric buildings, and those characteristics need to be transferred to a measure of distance representing how far away the equivalent lateral force should be applied to the center of mass to capture the same amount of torsional displacement amplification.

To analyze the displacement amplification caused by eccentric static loading, an eccentric equivalent lateral force is applied to the base system. In comparison, a non-eccentric equivalent lateral force is also applied to the base system right at the center of mass. The ratio between two displacements in these two scenarios demonstrates the amplification due to eccentric push over, as shown in Eq.1-4 and Eq.1-5, where *b* is the dimension of the plan perpendicular to the applied static force, and *e* is the eccentricity in percentage, *eb/2* is the distance from the applied force to the center of mass.

$$\alpha_p = \frac{\delta_p^e}{\delta_p^{ne}} = \frac{V/_{K_{trans}} + {}^{Ve}/_{K_{rot}} b/2}{V/_{K_{trans}}} \qquad \text{(Eq.1-4)}$$

$$V = S_a W/g \qquad \text{(Eq.1-5)}$$

An example of displacement amplification from eccentric push over analysis is shown in Figure 1-1: two buildings with plan aspect ratios of 1:1 and 1:8 are subject to eccentric lateral force, nine grey lines represent the linear relationship between displacement amplification and the eccentricity of the applied lateral force. Higher displacement amplification is seen in the building with higher plan aspect ratio, when *Ω* and lateral force eccentricity are both fixed.

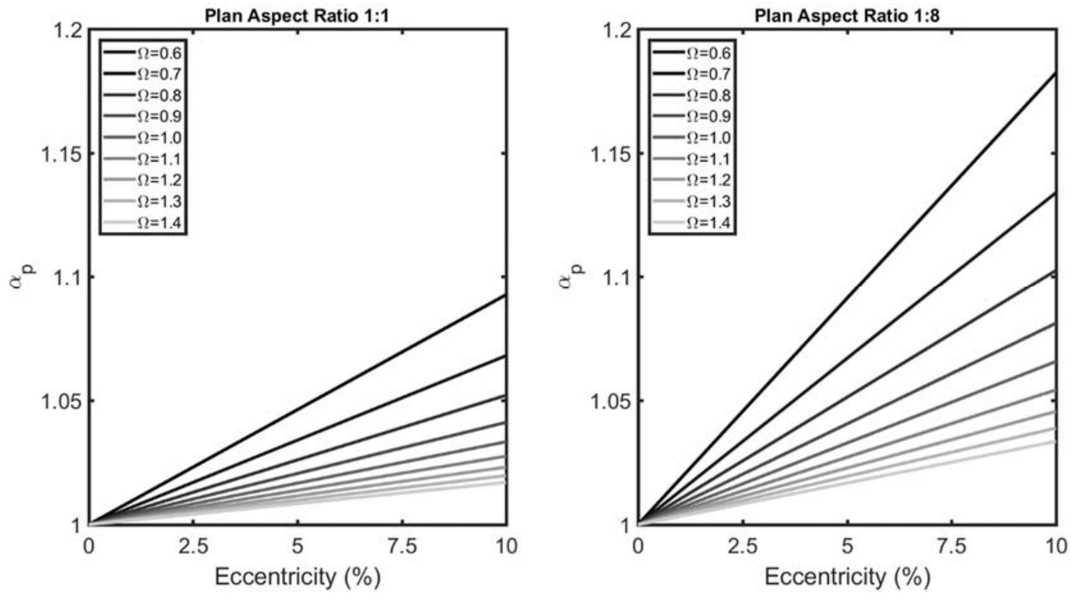

Figure 1-1. Amplification due to eccentric push over for nine base systems

α_1 , which is the ratio of the asymmetric system peak total response to the symmetric system peak translational response, is mapped to the eccentricity in percentage by equating average of α_1 and α_p. In other words, the equivalent eccentricity determines how far away should the equivalent static lateral force be applied from the center of mass to make the symmetric system have as much displacement amplification as an asymmetric system with uncertain VLLRs stiffness and subject to ground motions. Since each Monte Carlo simulation results in one α_1 value, there are 36000 realizations of α_1 for a given plan aspect ratio system. Statistical properties of α_1 database such as median and 75th percentile are mapped to the corresponding equivalent eccentricity, representing different levels of torsion design requirements.

Results

Results of equivalent eccentricities from Monte Carlo simulation are shown in Figure 2-1; observations and conclusions are as follows:

- 5% eccentricity from code provision is larger than the median response from the simulations, as the computed equivalent eccentricity of all systems with translational to rotational period ratio ranging from 0.6 to 1.4 and plan aspect ratio ranging from 1:1 to 1:8 fall below 5%.
- Higher levels of eccentricity (compared to 5%) is required to be applied to a system if confidence levels larger than 50% is of interest.
- Compared to buildings that have large rotational stiffness (*Ω* larger than 1), buildings that are sensitive to torsion (*Ω* less than 1) require less equivalent eccentricity at higher confidence level.
- Equivalent eccentricity (displacement amplification) is *Ω* sensitive. When translational period and rotational period are identical, it reaches its minimum value (almost equal to zero). Equivalent eccentricity displays an M shape over the *Ω* = 0.6 to 1.4 range.

- Buildings with larger plan aspect ratio do not necessarily have higher equivalent eccentricity, though they have relatively higher displacement amplification α_1 .

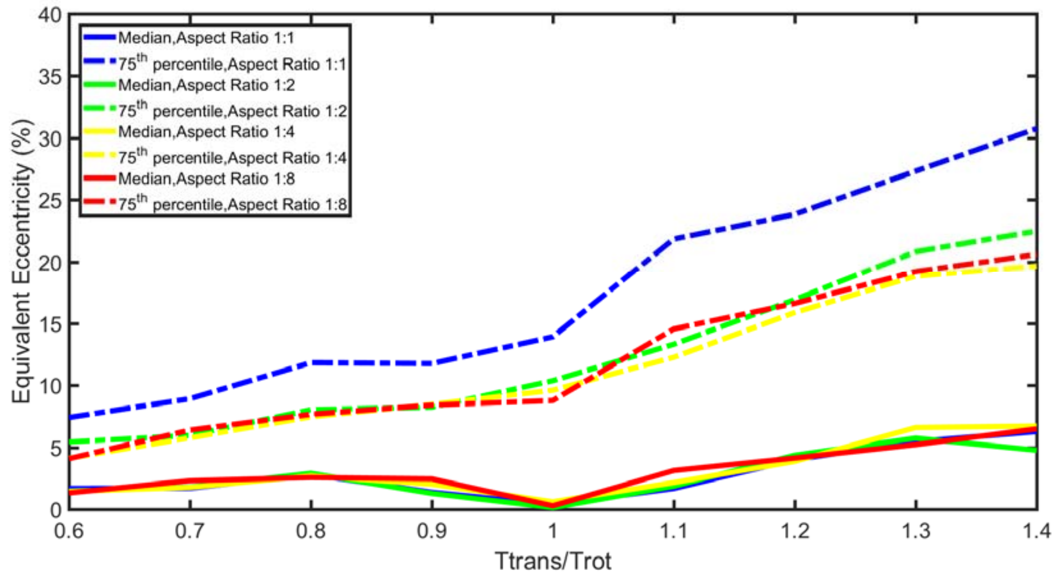

Figure 2-1. Equivalent eccentricity for buildings with *Ω* ranging from 0.6 to 1.4 and plan aspect ratio ranging from 1:1 to 1:8, using uncorrelated VLLRs stiffness model (ρ = 0)

The effect of VLLRs stiffness correlation on equivalent eccentricity is also studied; sample results are demonstrated in Figure 2-2 for a building with plan aspect ratio of 1:2 and ρ = 0.0 and 0.5. Using the median value of the simulations as the target displacement amplification to compute the equivalent eccentricity, systems with correlated VRRLs stiffness have smaller equivalent eccentricity and correspondingly smaller displacement amplification. This can be explained by the observation that when stiffness of the four VLLRs increase or decrease coherently (i.e. correlation), the level of asymmetricity is reduced and leads to a reduction in displacement amplification. Asymmetricity can be estimated by the off-diagonal element (Eq.2-1) in equation of motion, where *D* is the perpendicular distance between center of mass and VLLRs, k_1 and k_2 are stiffness of VRRLs along X axis.

$$k_{x\theta} = \sum k_i x_i = (k_1 - k_2)D \qquad \text{(Eq.2-1)}$$

Variation in difference between the stiffness of VLLRs decreases when introducing correlation to VLLRs stiffness as shown in Eq. 2-2.

$$\sigma^2_{k_1-k_2} = \sum_{i=1}^{2}\sum_{j=1}^{2}\rho_{ij}\sigma_{k_i}\sigma_{k_j} = \sigma^2_{k_1} + \sigma^2_{k_2} - 2\rho\sigma_{k_1}\sigma_{k_2} \qquad \text{(Eq.2-2)}$$

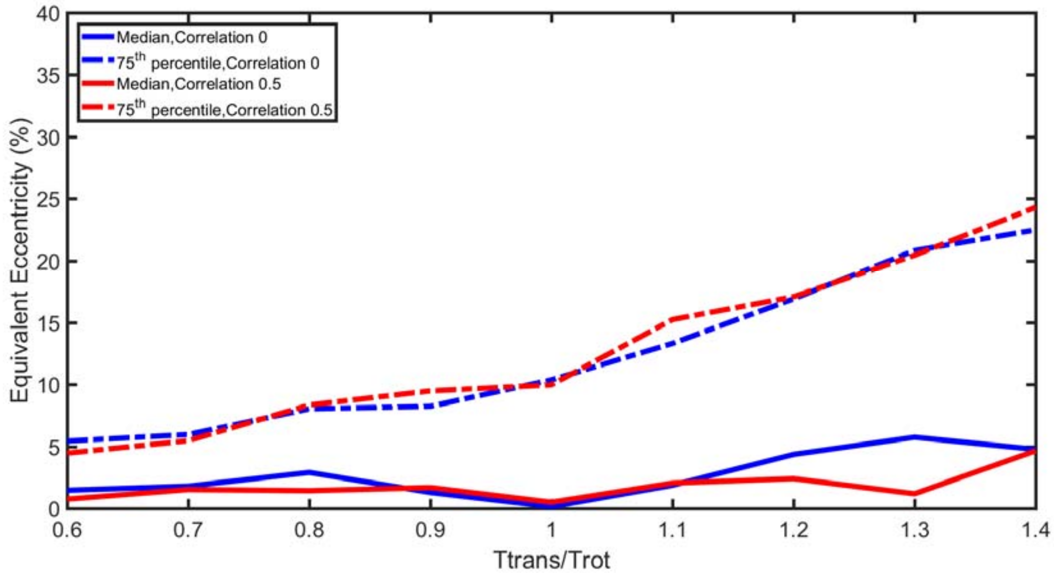

Figure 2-2. Equivalent eccentricity for buildings with *Ω* ranging from 0.6 to 1.4 and two stiffness correlation (ρ = 0, ρ = 0.5), using a plan aspect ratio 1:2 model

The difference between equivalent eccentricity of systems with and without VLLR stiffness correlation diminishes at large quantiles (see Figure 2-2 for the 75% quantile). This numerical issue may happen due to the small amount of total simulation numbers to capture equivalent eccentricity at the tail of its distribution. Nevertheless, equivalent eccentricity associated with uncorrelated stiffness cases are preferred since it provides a more conservative estimate of accidental torsion.

The distribution of three torsional vibration characteristics (α_1 , α_2 and α_3) are plotted in forms of box plots and compared among four systems with different plan aspect ratios. Each box plot shows five quantiles of the data set: top and bottom sides of the blue box show 75 and 25 percentiles; the red bar in the middle of the blue box shows the median; and top and bottom whiskers show an extension equal to 1.5 times the difference between the values associate with 75 and 25 percentiles to the 75 and 25 percentile values respectively. Figure 2-3, Figure 2-4 and Figure 2-5 show the boxplots for α_1 , α_2 and α_3 , respectively. The following observations are drawn from Figures 2-3, 2-4, and 2-5:

- The total amplification is mainly due to translational displacement other than rotational displacement, since no large difference is observed between α_1 and α_3 , and the mean value of α_2 is below 1.05. That reveals the fact that amplification in translational response due to simultaneous decrease in VLLRs stiffness affects the building more than amplification in rotational response due to the difference in stiffness of the opposite VLLRs.
- An increase in plan aspect ratio results in an increase in displacement amplification due to uncertainty in stiffness.

- Buildings with plan aspect ratio of 1:1 have smaller rotational amplification (α_2) compared to other plan aspect ratios.
- In average, total displacement amplification (α_1) is less than 1.05, and is minimized when the building's translational period equals its rotational period.
- Large variance of three vibration characteristics show the importance of extreme cases. Design for confidence levels larger than 50% would result in large values of α_1 , α_2 and α_3 .

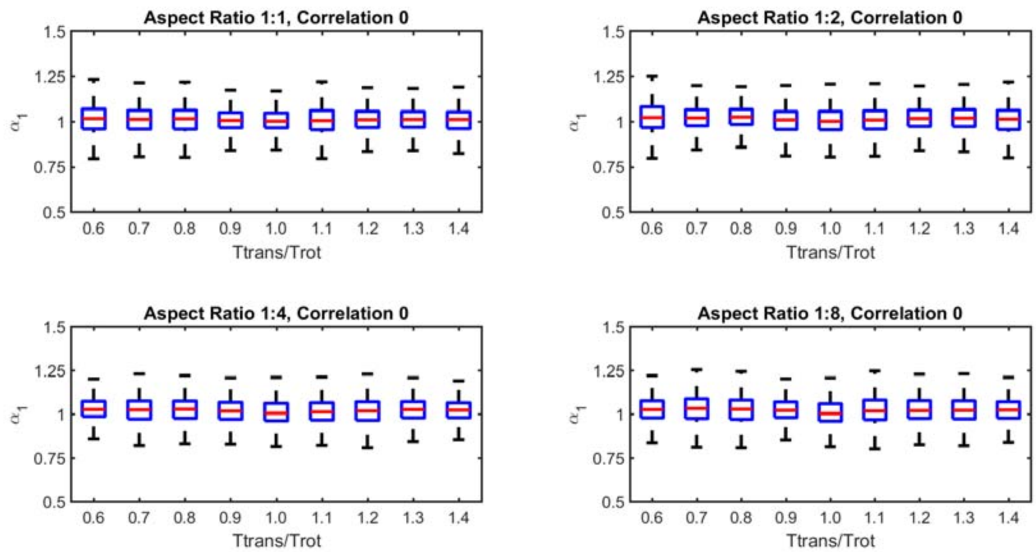

Figure 2-3. Distribution of α_1 for buildings with Ω ranging from 0.6 to 1.4 and plan aspect ratio ranging from 1:1 to 1:8

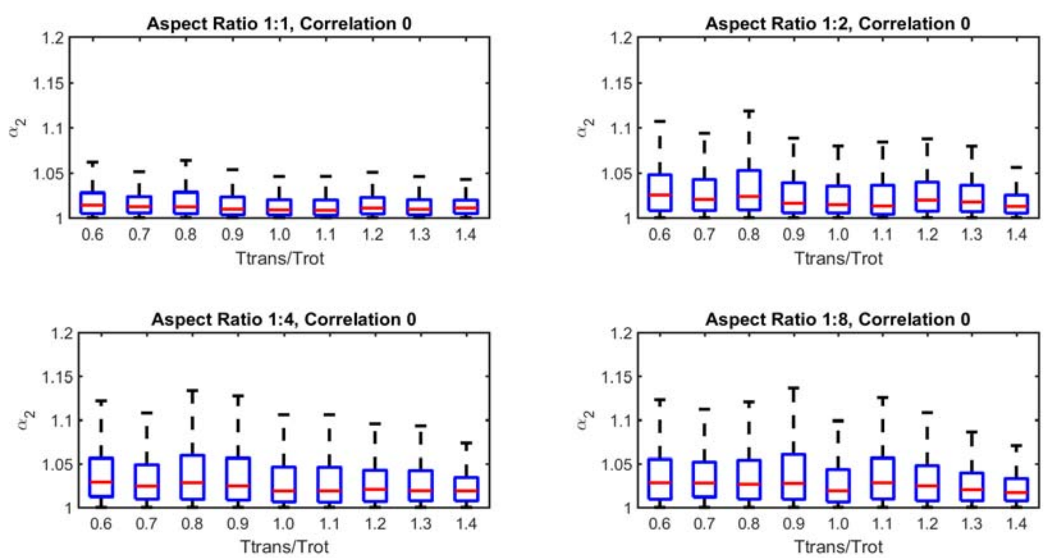

Figure 2-4. Distribution of α_2 for buildings with Ω ranging from 0.6 to 1.4 and plan aspect ratio ranging from 1:1 to 1:8

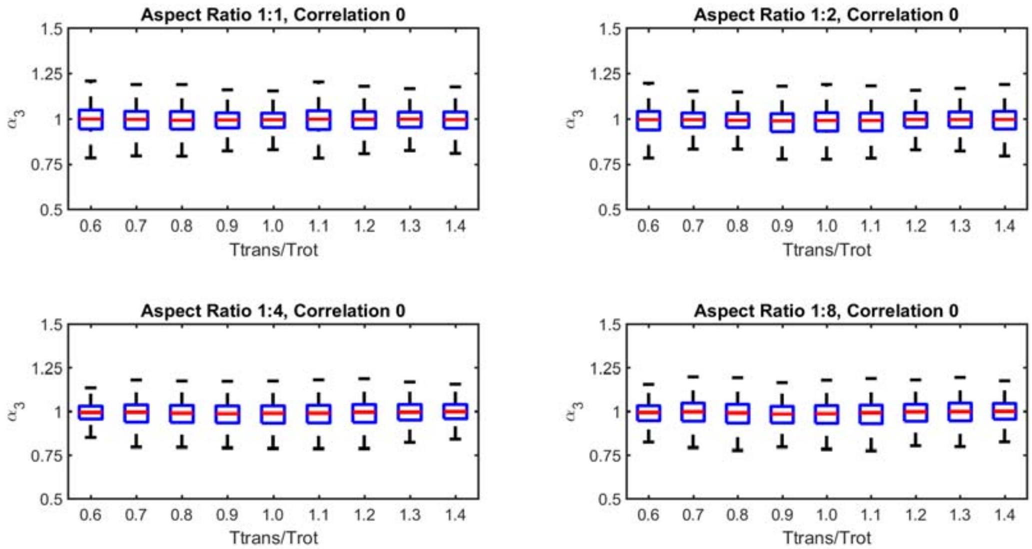

Figure 2-5. Distribution of α_3 for buildings with Ω ranging from 0.6 to 1.4 and plan aspect ratio ranging from 1:1 to 1:8

Validity of simulation results are checked using records from CSMIP database. Three-dimensional system identification techniques (Juang,1997, Van Overschee, 1996, Zhang 2001) are applied to four selected SMRF buildings (combinations of two plan aspect ratio of 1:1 and 1:2, and two levels of building height: high-rise and low-rise) in the CSMIP database to obtain translational to rotational period ratio. α_2 , the ratio of peak total response to peak translational response within an asymmetric system is computed in each of the four selected buildings. Since results of computed displacement amplification can be inaccurate at high noise levels, only those records with PGV (peak ground velocity) larger than 5cm/s are selected. Building information can be found in Table 2-1 and Figure 2-6.

Table 2-1. Selected buildings and ground motions information

Station	Height (ft)	Aspect Ratio (X to Y)	Ground Motion	PGV_x PGV_y (cm/s)	α_{2x} α_{2y}	Ω_x Ω_y
14533	265	1:1	Whittier 87	6.86	1.03	1.03
				4.43	1.07	1.11
23516	41.3	1:1.1	Calexico 04Apr 2010	2.50	1.01	1.22
				5.85	1.01	1.28
24104	41	1:2	Calexico 04Apr 2010	3.08	1.01	1.37
				1.80	1.01	1.36
24569	274	1:2.1	Landers 92	7.63	1.06	0.93
				12.42	1.02	0.96

Figure 2-7 shows the vibrational characteristic α_2 versus translational to rotational period ratio Ω of four selected buildings from CSMIP database superposed on the corresponding simulation results. It is notable that due to lack of records for one-story building, none of the selected buildings are one-story systems as the building models used for simulation purpose. However, Chopra (1995) showed that displacement amplification in a multistory building can be approximated by a single-story system with the same Ω as long as it satisfies: 1) The centers of mass of all floors lie on a vertical line; 2) Resisting planes form an orthogonal grid and are connected by a rigid diaphragm at each floor; 3) Lateral stiffness matrices of all resisting frames are proportional to each other. Thus, the displacement amplification of four selected symmetric-in-plan buildings can be approximated by their one-story counterparts.


14533


23516


24104


24569

Figure 2-6. Selected buildings from CSMIP database

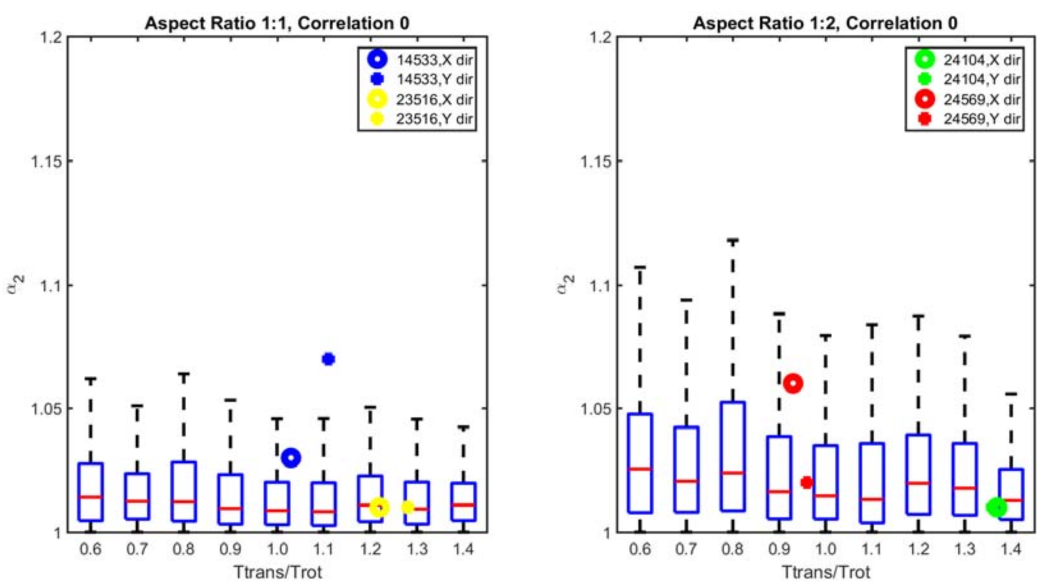

Figure 2-7. Comparison between simulation results and records from four selected buildings in CSMIP database

It can be observed from Figure 2-7 that displacement amplification of low-rises (building ID 23516 and 24104) are close to the median of simulation results. Simulations used in this research take into account VLLRs stiffness uncertainty only, while real-life data contains sources other than stiffness uncertainty that may cause torsional vibration (e.g. uncertainty in mass and location of center of mass). Nevertheless, displacement amplification in low-rise buildings has a good match with simulation results at median level. In high-rises (building ID 14533 and 24569), however, data leans towards higher than 75^{th} percentile amplification in displacement, and more extreme cases can be observed. This shows the need for extending this investigation into models other than one-story systems to study the torsional behavior of high-rises.

Summary and Future work

This research develops statistical information on building vibrational characteristics such as displacement amplification factors and equivalent eccentricity using Monte Carlo simulations of one-story systems. Simulation results are verified using records from CSMIP database. Building properties such as plan aspect ratio, translational to rotational period ratio, correlation between VLLRs stiffness are found to be of great importance for prediction of torsional behavior of a building. Conclusions of this study are as follows:

- Displacement amplification due to torsion is highly affected by Ω. Buildings with a translational period identical to rotational period (Ω =1.0) and buildings who are insensitive to torsion (Ω >1.4) tend to have smallest amplification and are least affected by torsional vibration.
- An increase in plan aspect ratio results in an increase in displacement amplification.
- Correlation between VLLRs stiffness reduces displacement amplification due to torsion.
- To account for accidental torsion, the 5% rule is higher than how much an equivalent eccentricity requires at a median level. But when higher confidence level is preferred, equivalent eccentricity can be larger than 5%.

This study mainly focuses on torsional effect of one-story symmetric-in-plan linear system due to uncertainty in stiffness. Aside from plan aspect ratio and period ratio, building height could be one predictor of displacement amplification (as is demonstrated in Figure 2-7). In future studies, building height and nonlinear behavior will be studied. 4-story, 8-story, 12-story and 20-story building models with bilinear hysteretic materials are built to take into account stiffness and strength uncertainty. These building models can also capture the effect of number of VLLRs along one direction.

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