

# SMIP90 Seminar Proceedings

## STRENGTH AND DUCTILITY DEMANDS FOR SDOF AND MDOF SYSTEMS SUBJECTED TO WHITTIER NARROWS EARTHQUAKE GROUND MOTIONS

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### ABSTRACT

This paper summarizes parts of a project that is concerned with an evaluation of the damage potential of the October 1, 1987 Whittier Narrows ground motions. The discussion focuses on strength and ductility demands imposed by these ground motions on single degree of freedom (SDOF) and multi-degree of freedom (MDOF) systems. In the SDOF study, bilinear as well as stiffness degrading models are evaluated. A procedure is presented that permits the evaluation of strength demands for MDOF systems based on inelastic strength demand spectra of SDOF systems.

### INTRODUCTION

Proper seismic design implies that strength and deformation capacities of structures should exceed the demands imposed by severe earthquakes with an adequate margin of safety. This simple statement is difficult to implement because both demands and capacities are inherently uncertain and dependent on a great number of variables. Moreover, more knowledge has to be acquired on the relative importance of different demand and capacity parameters and on the methods of incorporating these parameters in design without unduly complicating the design process. For these and many other reasons, code design is presently based on simplified procedures that have no transparent relation to many of the demand and capacity parameters that dominate the seismic response of structures.

A desirable long-range objective of research in earthquake engineering is to provide the basic knowledge needed to permit a simple but explicit incorporation of relevant demand and capacity parameters in the design process. To this end, much work needs to be done. Identification and evaluation of relevant ground motion and seismic demand parameters is one aspect of this work. To this date the earthquake engineering profession has not succeeded in identifying and quantifying a set of ground motion and seismic demand parameters that is sufficiently complete, yet simple enough, to permit capacity/demand based code design. In this context, a demand parameter is defined as a quantity that relates seismic input (ground motion) to structural response. Thus, it is a response quantity, obtained by filtering the ground motion through a linear or nonlinear structural filter. A simple example of a demand parameter is the acceleration response spectrum, which identifies the strength demand for an elastic SDOF system. Considering that most structures behave inelastically in a major earthquake, it is evident that this parameter alone is insufficient to describe seismic demands.

The Whittier Narrows earthquake provided an extensive set of ground motion records and damage information. It is the objective of this study to utilize the CSMIP ground motion data to evaluate basic seismic demand parameters, search for patterns in their characteristics, study the

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attenuation of the parameters, and use the demand information together with capacity information on several types of code designed structures to assess the damage potential of the Whittier Narrows ground motions. The early results of this study have been summarized in [Krawinkler and Nassar, 1989]. The discussion presented here focuses an evaluation of strength and ductility demands, with an emphasis on MDOF systems. In order to put this issue in perspective it is necessary first to identify the objectives of demand and capacity based seismic design, a design approach that shows much potential for improvement of present design practice.

### DEMAND AND CAPACITY BASED SEISMIC DESIGN

Good seismic design implies that structural capacities should exceed demands imposed by earthquakes with a sufficient margin of safety to provide adequate protection, with due consideration given to the uncertainties inherent in demands and capacities. Demands are response quantities that affect capacities and can be computed from ground motion and structural system characteristics, and include, amongst others, elastic strength demand,  $F_{y,e}$ , inelastic strength demands,  $F_y(\mu)$ , ductility demand,  $\mu$ , strength reduction factors,  $R_y(\mu) = F_{y,e}/F_y(\mu)$ , and cumulative damage demands, of which energy demands are a subset. Definitions of these terms are given in [Krawinkler and Nassar, 1989].

Capacities are limiting values of the same response quantities, employed to assure adequate safety against failure. From the aforementioned quantities, ductility, or more specific, local or element ductility, appears to be the fundamental capacity parameter. If an element fails in a non-ductile mode (e.g., connections, or columns under axial compression), its ductility capacity is 1.0 or smaller. If an element exhibits ductile behavior (e.g., flexural hinging in beams), its ductility capacity is larger than 1.0. The magnitude of the ductility capacity of an element is a matter of structural behavior and will depend also on cumulative damage demands. Explicit consideration of cumulative damage demands (e.g., hysteretic energy dissipation) in design is possible but may unduly complicate the design process. Due consideration can be given to these demands by modifying the ductility capacity for strong motion duration, energy dissipation demands, or other cumulative damage parameters.

If ductility is used as a basic capacity parameter for design, then strength becomes a dependent quantity than can be derived from the criterion that ductility capacity must exceed ductility demand. This is believed to be the intent of code design, but the implementation in present codes is implicit and nontransparent. Explicit consideration of ductility capacity in design is feasible, as has been pointed out in many recent, and some old, technical papers. A framework for ductility based design has recently been proposed [Osteraas and Krawinkler, 1990]. Implementation of this design procedure requires, amongst other, much more information on seismic demands.

The Whittier Narrows earthquake has provided an extensive ground motion data set from a single earthquake, permitting statistical studies as well as an evaluation of attenuation of relevant seismic demand parameters. A total of 33 CSMIP ground motions recorded in this earthquake are used in this study to evaluate these parameters for SDOF and MDOF systems.

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## SEISMIC DEMANDS FOR SDOF SYSTEMS

A great number of demand parameters have been investigated in this study, with some of the results summarized in [Krawinkler and Nassar, 1989] and presented, wherever possible, in terms of spectra that vary with epicentral distance. Initially the analysis was performed with bilinear SDOF systems with 10% strainhardening and 5% damping. Recently the analysis was repeated with stiffness degrading systems in order to evaluate the effect of stiffness degradation on important demand parameters. A typical comparison of strength demands for bilinear and stiffness degrading systems is shown in Fig. 1. The stiffness degradation model used in this analysis was a modified Clough model illustrated in Fig. 2.

The general conclusion from this part of the study is that stiffness degradation of the type represented by the modified Clough model does not have a dominant effect on most of the important demand parameters. This can be seen in the example presented in Fig. 1. The illustrated inelastic strength demand spectra for  $\mu = 2, 3, \text{ and } 4$ , which apply for an epicentral distance of 10 km and are developed from a regression analysis, differ very little between the bilinear and stiffness degrading model. Comprehensive results of the evaluation of SDOF seismic demands will be presented soon in a report to be submitted to CDMG.

## EFFECT OF HIGHER MODES ON SEISMIC DEMANDS FOR MDOF SYSTEMS

The previous discussion has focused on the nonlinear dynamic response of SDOF systems. Unfortunately, most real structures are MDOF systems affected by several translational and torsional modes. For elastic MDOF systems, combination of modal responses using SRSS, CQC, or other approaches, provides reasonable estimates of peak dynamic response characteristics. For inelastic MDOF systems, modal superposition cannot be applied with any degree of confidence. Thus, it becomes a matter of much interest to find out if and how the demand predictions for SDOF systems can be applied to MDOF structures.

In the pilot study summarized here it is assumed that only translational response needs to be considered; torsion is neglected. Thus, structures are modeled two-dimensionally. A series of structures are designed and subjected to the 33 ground motions used in the SDOF response study. Of primary interest are strength and ductility demands since they are basic design parameters that can be compared directly to counterparts in the SDOF study.

The following assumptions are made in the design of the MDOF structures:

- Only structures with 2, 5, 10, 20, 30, and 40 stories are considered. Equal mass is assumed for each story and the height of each story is taken as 12 ft.
- Structures are modeled as single bay frames and are of one of the following two types: "Column Hinge" model, from here on called CH model (Fig. 3(a)), or "Beam Hinge" model, from here on called BH model (Fig. 3(b)). A clear distinction is made between these two models, as significant differences in the response are expected. The CH model is used to model braced frames (or moment frames with column plastic hinges) in which story mechanisms can develop; the BH model is used to model structures in which mechanisms involve the full structure.

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- The strengths of the members are tuned in a manner so that mechanisms develop simultaneously in every story under the UBC88 seismic lateral load pattern. Thus, under this load pattern the lateral load - lateral deflection response is bilinear. A strain hardening ratio of 10% is assumed at each plastic hinge location, resulting in a bilinear load deflection response that resembles that of the bilinear SDOF model without stiffness degradation, which was employed as one of the models of the SDOF study.
- The member stiffnesses in each story are tuned in a manner so that under the UBC88 seismic load pattern the interstory drift in every story is identical, resulting in a straight line deflected shape. As a consequence, the first mode shape of all structures is close to a straight line. Furthermore, the stiffnesses are tuned in a manner so that the first mode period of each structure is equal to the UBC88 code period of  $0.002h_n^{2/3}$  for braced frames. This tuning is done also for the BH model in order to permit a direct comparison of dynamic analysis results between CH and BH models. Pertinent data for the first five elastic modes of the CH models are presented in Table 1. The periods for the BH models are the same for the first mode but deviate slightly for higher modes because of difficulties in stiffness tuning. As a consequence the modal masses also differ slightly. For the first two modes a damping of 5% critical is assumed in the time history analysis.
- The base shear strength,  $V_y$ , is varied for each structure and ground motion record in a manner so that it is identical to the inelastic strength demand  $F_y(\mu)$  of the corresponding first mode period SDOF system for ductilities of either 2, 3, or 4.

The last assumption is critical for the intended comparison of SDOF and MDOF seismic demands. Because modal superposition is hardly feasible for inelastic MDOF systems, it is desirable to utilize SDOF demand estimates to predict MDOF demands. The inelastic strength demands for target ductilities can be evaluated for SDOF systems as discussed in [Krawinkler and Nassar, 1989], with typical results shown in Fig. 1. The question to be answered is how different the ductility demands for the MDOF systems are if the base shear strength of the MDOF system is identical to the inelastic strength demand of the SDOF system for the prescribed ductility. Or even better, the question to be answered is how large should the base shear strength of the MDOF system be [assuming the code prescribed seismic load pattern] in order to limit the maximum ductility in the MDOF system to a certain prescribed value. The results discussed next provide a partial answer to these questions.

The model structures, with their strength tuned as discussed in the preceding paragraphs, were subjected to the 33 CSMIP ground motions using a modified version of the DRAIN-2D analysis program. Results for mean values of ductilities obtained from the analyses with the 33 records are illustrated in Figs. 4 to 7.

Figure 4 shows the means of the story ductilities for each story of CH models with a base shear strength  $V_y$  equal to the inelastic strength demand  $F_y(2)$  corresponding to a ductility of 2 for the SDOF system with a period equal to the first natural period of the MDOF structure. From here on the SDOF ductility is called the target ductility. If the MDOF system would respond like a SDOF system, the story ductility would also be 2 in every story. Clearly this is not the case, as expected, because of higher mode participation. For structures of ten stories and less the deviations from the SDOF target ductility of 2 are moderate, for taller structures the deviations

become very large. These large deviations for tall structures come as no surprise as the Whittier Narrows ground motions had very low energy content at periods exceeding one second and, therefore, the inelastic strength demand  $F_y(\mu)$  for long period SDOF systems, which is used as base shear strength for the MDOF systems, is very small. As a consequence, higher mode effects become dominant for tall structures subjected to Whittier Narrows ground motions, more so than for other ground motions. Because of this peculiarity of the Whittier Narrows ground motions, no general conclusions can or should be drawn on seismic demands for long period structures, and emphasis from here on is on 2 to 10 story structures.

Figure 5 shows the means of the story ductilities for CH models with a target ductility of 4 (i.e., these structures are designed for a base shear strength  $V_y = F_y(4)$ ), and Figures 6 and 7 show the results for target ductilities of 2 and 4, respectively, for BH models with 2, 5, and 10 stories. At the time of writing, the results for the 20, 30, and 40 story BH models are not yet available. The results in Figures 4 to 7 show consistent trends that can be summarized as follows:

- Except for the 2-story models, the story ductility demands are higher than the target ductility in the bottom and top stories, and are close to or smaller than the target ductilities in the middle stories. Thus, the UBC88 seismic design load pattern does not create an equal ductility demand for all stories (it probably was not intended to do so); ductility demands are concentrated in top and bottom stories.
- For the five and 10 story buildings the story ductility demands are higher in the top story than in the bottom story even though all stories yield simultaneously under static loads conforming to the code seismic load pattern. Thus, if the strength of all stories is fine-tuned to the code seismic load pattern, high upper story ductilities have to be expected. This indicates that a closer look should be taken at the presently used seismic load pattern.
- The ductility demands for the CH model is consistently higher than the demands for the BH model even though both models are designed for the same base shear strength. This is more evident from Figs. 8 and 9, which show the maximum ductility demands for all stories and the ductility demands for the first story, respectively. Again, each presented value is the mean obtained from the results of 33 time history analyses. The disadvantage of the CH models, in which individual story mechanisms can form, is evident. However, for the range of direct comparison, i.e., 2 to 10 story buildings, the difference between the two models is not as overpowering as often argued in defense of the need for strong column - weak girder design.

Figures 8 and 9 illustrate also the differences between SDOF and MDOF ductility demands. The base shear strength of the MDOF systems,  $V_y$ , was taken equal to the inelastic strength demand,  $F_y(\mu)$ , of the SDOF system with the period of the fundamental MDOF mode and ductilities of  $\mu = 2, 3, \text{ or } 4$ . Thus, values of  $\mu = 2, 3, \text{ and } 4$  are target ductilities, and deviations from these targets are due to higher mode effects. As the two figures show, the deviations exhibit consistent trends, being small for two story CH and BH models, moderate for five and ten story CH and BH models, and becoming large for CH models of more than ten stories. Since the ductility demands for the MDOF systems are usually larger than the target ductilities, the implication is that the base shear strength of the MDOF systems must be increased in order to limit the ductility demand to the desired target values. The question is by how much. The following discussion provides preliminary answers.

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For SDOF systems, the parameter that relates elastic to inelastic strength demands is the strength reduction factor  $R_y(\mu) = F_{y,e}/F_y(\mu)$ . It is a convenient design parameter as it permits, in concept, the derivation of a design strength spectrum from an acceleration response (elastic strength demand) spectrum. Although it is arguable whether it is the right parameter for this purpose, it is used here to address the question posed. From the regression analysis performed with the 33 Whittier Narrows records used in this study it was found that the strength reduction factor is not very sensitive to epicentral distance, thus, this factor can be represented by distance independent mean and mean+ $\sigma$  values. These values, plotted versus period, are shown in Fig. 10. The figure shows trends similar to those reported by others but the R-factors are larger than is usually assumed, an issue that is not important in the context of this discussion.

The advantage of using strength reduction factors, which are strength demand ratios, is that they are dimensionless parameters that do not depend on the severity of ground motions and can be utilized to evaluate relative strength demands. Their utilization for the 2, 5, and 10 story model structures is illustrated in the  $R_y(\mu)$  versus  $\mu$  plots shown in Figs. 11 to 13. The points shown on each graphs are obtained by using, as ordinates, the mean strength reduction factors for  $\mu = 2, 3,$  and  $4$  of the SDOF system with the period of the first mode of the MDOF system (these values define the relative strength of the SDOF and MDOF models), and as abscissa, the ductilities of the SDOF and MDOF models. Connecting the three data points for each model with straight lines gives approximate  $R$ - $\mu$  relationships for the SDOF as well as MDOF models. Figures 11 to 13 show that for 2-story buildings the  $R$ - $\mu$  relationships for MDOF systems are very close to that of the SDOF system, but they deviate by increasing amounts as the number of stories increases.

Within the limitation of the approximate nature of the straight-line interpolation, Figs. 11 to 13 provide the information needed to assess the required increase in strength for MDOF systems compared to the SDOF system of first mode period, in order to limit the story ductility in the MDOF systems to the target value of the SDOF system. For instance, the R-factor for the 10-story BH model with a target ductility of 3 is approximately equal to 3.5, whereas the R-factor of the SDOF system is 4.2. Thus, the base shear strength of the 10-story BH model should be increased by a factor of about  $4.2/3.5 = 1.2$  compared to the inelastic strength demand  $F_y(3)$  of the SDOF system. If the less desirable CH model is used, this factor becomes about  $4.2/3.2 = 1.3$ .

The foregoing discussion points out a procedure that can be employed to derive design strength demands for MDOF systems from inelastic strength demand spectra of SDOF systems. The few numerical results shown cannot be generalized without a much more comprehensive parameter study. The parameters that need to be considered include the frequency content of the ground motions (which may be greatly affected by local site conditions), the hysteretic characteristics of the structural models (strainhardening, stiffness and strength degradation, etc.), and the dynamic characteristics of the MDOF models (periods, mode shapes, and modal masses of all important modes). Furthermore, it must be pointed out that the  $R$ - $\mu$  relationships for MDOF systems developed here are based on ductility demands in the first story. In several cases the maximum ductility does not occur in the first story, but at the top of the structures. This issue is not considered here, but can be addressed through modifications of the distribution of design story forces over the height for structures without stiffness discontinuities (regular structures) and through dynamic analysis studies for irregular structures.

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## CONCLUSIONS

In the context of long range research objectives directed towards improvements of seismic design, the following conclusions can be drawn:

- Consistent protection against failure requires that ductility capacity, which may be modified for cumulative damage effects, becomes the basic design parameter.
- The strength of structures is a dependent quantity that depends on the specified ductility capacities.
- For SDOF systems the required strength for specified ductilities can be evaluated from statistical studies of ground motion response of representative hysteretic systems, and can be represented in terms of inelastic strength demand spectra.
- For MDOF systems the required base shear strength must be modified compared to the SDOF inelastic strength demand to account for higher mode effects. A procedure for implementing this modification is presented in this paper.

## ACKNOWLEDGEMENTS

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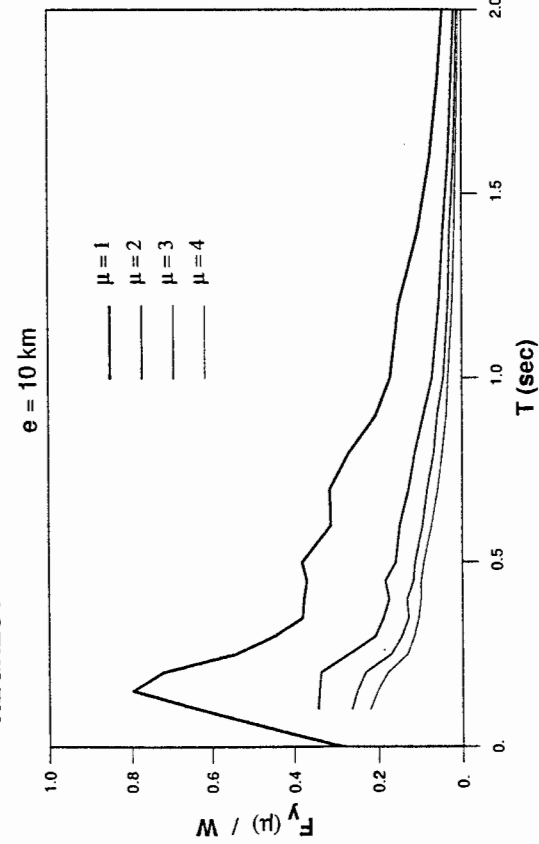
## REFERENCES

1. Krawinkler, H., and Nassar, A., 1989. "Damage Potential of the Whittier Narrows Earthquake Ground Motions," SMIP89, Proceedings of the Seminar on Seismological and Engineering Implications of Recent Strong-Motion Data, California Department of Conservation, Sacramento, California, May 1989.
2. Osteraas, J., and Krawinkler, H., 1990. "Strength and Ductility Considerations in Seismic Design," John A. Blume Earthquake Engineering Center Report, Department of Civil Engineering, Stanford University, to be published in June 1990.

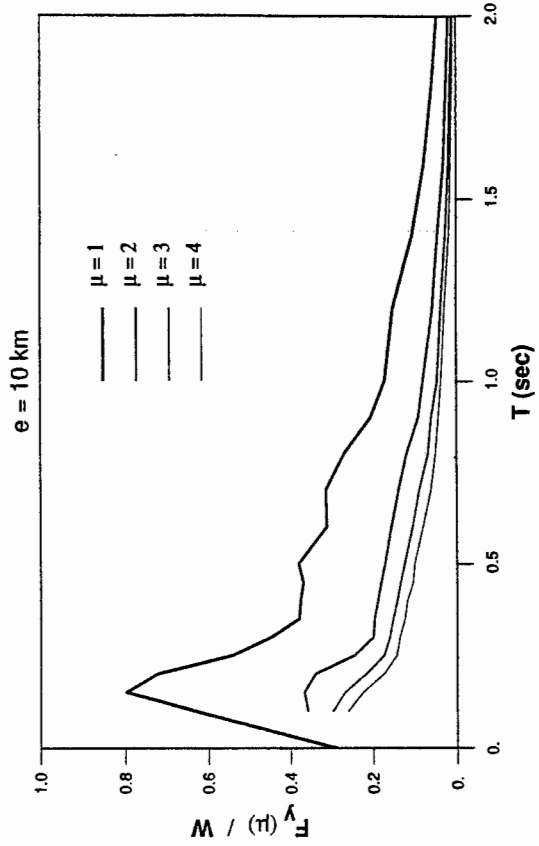
Table 1. Modal Periods and %Mass for Column Hinge (CH) Models

MODE	2-STORY		5-STORY		10-STORY		20-STORY		30-STORY		40-STORY	
	T (sec)	% MASS	T (sec)	% MASS	T (sec)	% MASS	T (sec)	% MASS	T (sec)	% MASS	T (sec)	% MASS
1	0.217	90.0	0.431	81.8	0.725	79.4	1.220	78.1	1.653	77.8	2.051	77.8
2	0.089	10.0	0.176	11.4	0.288	11.2	0.475	11.0	0.636	10.9	0.781	10.8
3			0.111	4.1	0.181	4.1	0.294	4.1	0.391	4.1	0.479	4.0
4			0.082	1.9	0.133	2.1	0.214	2.1	0.282	2.1	0.345	2.1
5			0.064	0.8	0.106	1.2	0.169	1.3	0.221	1.3	0.269	1.3

REGRESSED STRENGTH DEMAND - BILINEAR MODEL



REGRESSED STRENGTH DEMAND - STIFF. DEGRADING MODEL



(a) Bilinear Model

(b) Stiffness Degrading Model

Fig. 1 Elastic and Inelastic Strength Demand Spectra for Bilinear and Stiffness Degrading SDOF Systems

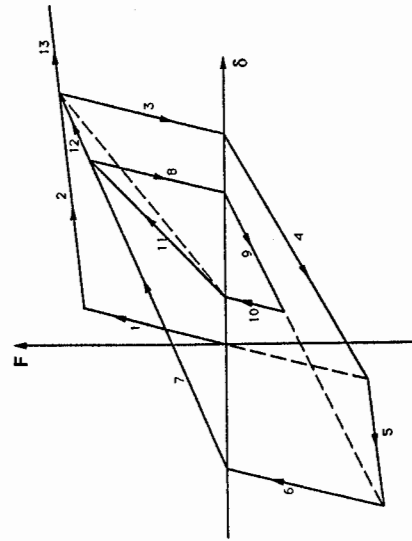
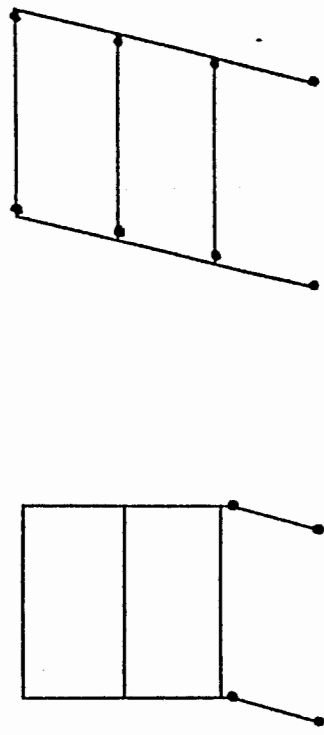


Fig. 2 Modified Stiffness Degrading Clough Model



(a) Column Hinge (CH) Model (b) Beam Hinge (BH) Model

Fig. 3 Types of Structures Used in MDOF Study



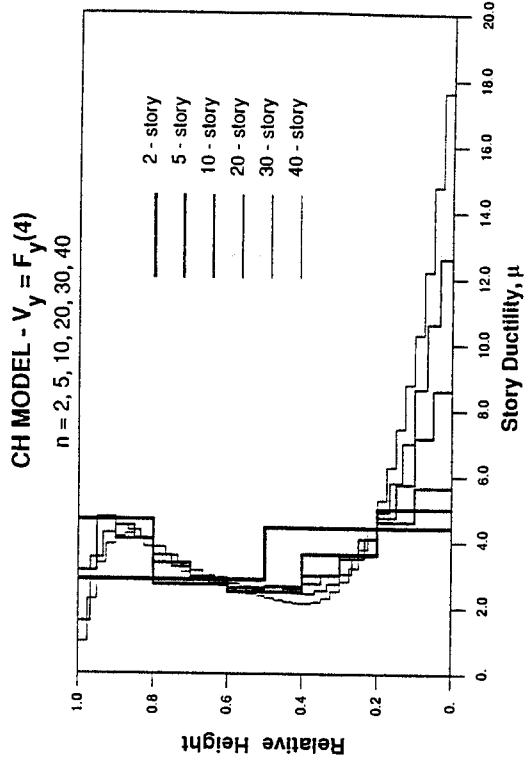


Fig. 5 Story Ductilities for CH Models with Target Ductility of 4

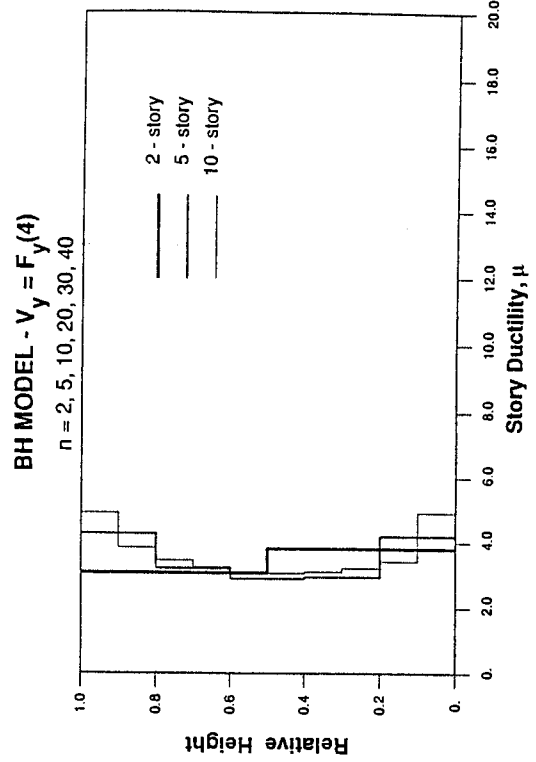


Fig. 7 Story Ductilities for BH Models with Target Ductility of 4

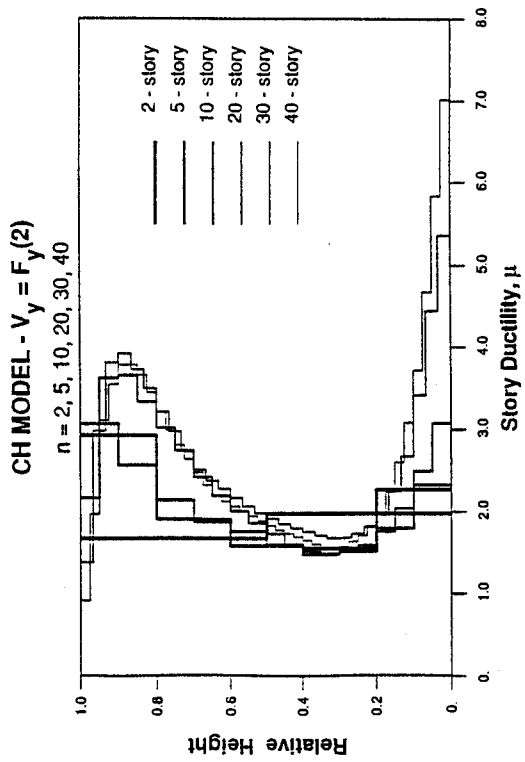


Fig. 4 Story Ductilities for CH Models with Target Ductility of 2

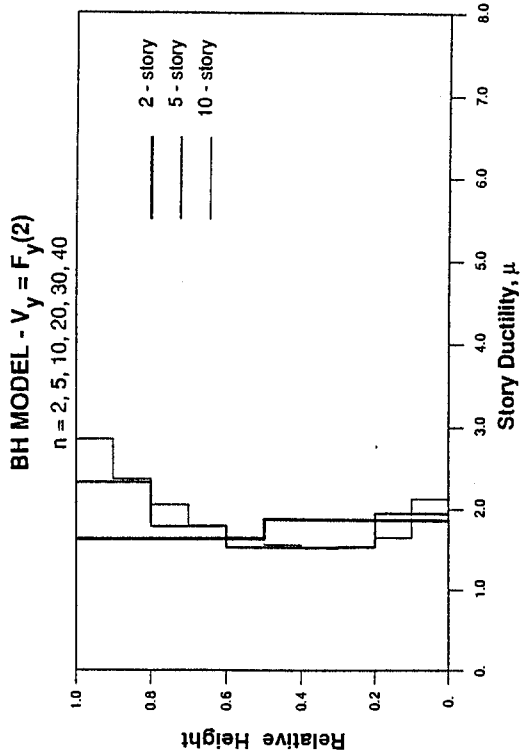


Fig. 6 Story Ductilities for BH Models with Target Ductility of 2

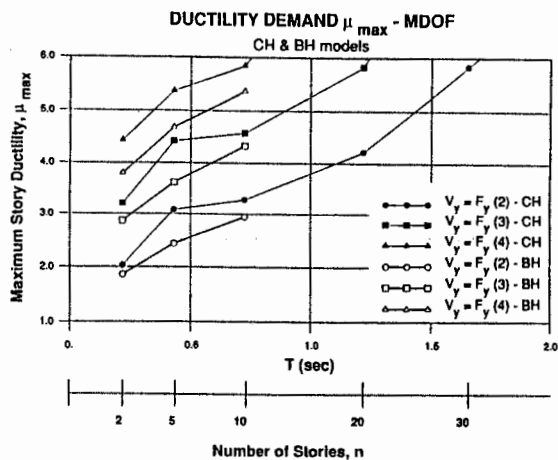


Fig. 8 Max. Story Ductilities for CH and BH Models

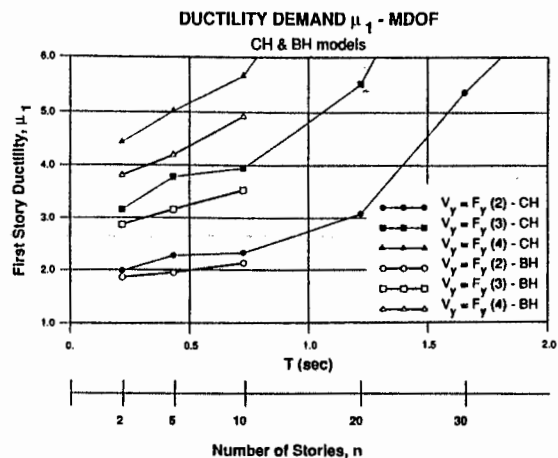


Fig. 9 Bottom Story Ductilities for CH and BH Models

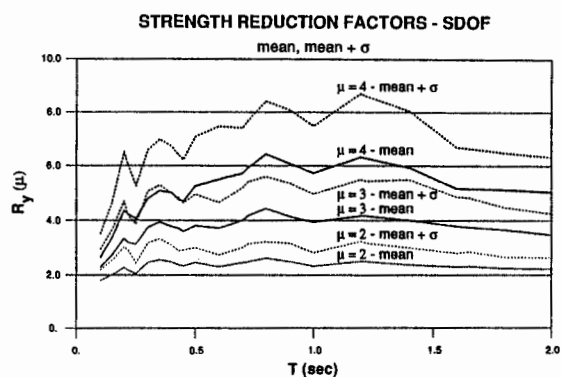


Fig. 10 Strength Reduction Factors for Bilinear SDOF Systems

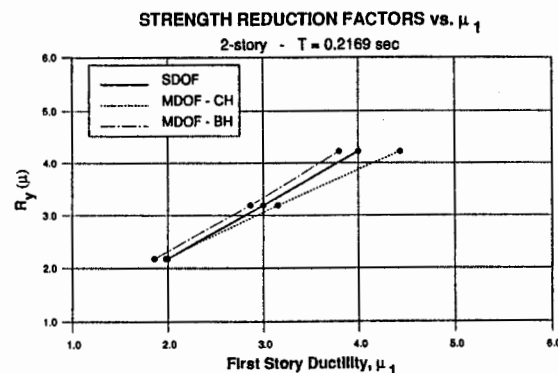


Fig. 11 R-μ Relationships for 2-Story CH and BH Models and Corresponding SDOF System

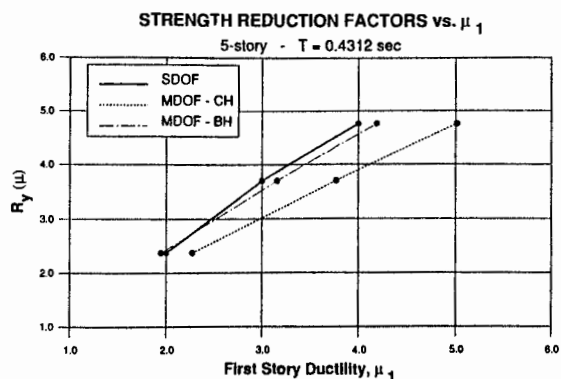


Fig. 12 R-μ Relationships for 5-Story CH and BH Models and Corresponding SDOF System

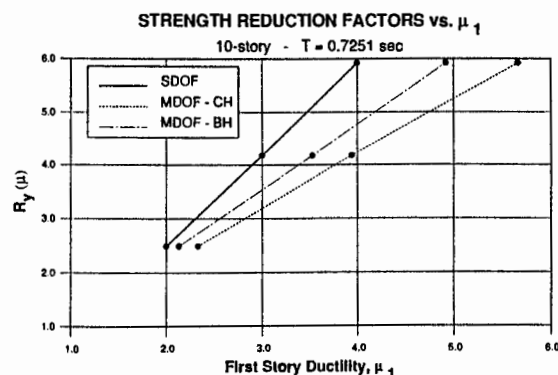


Fig. 13 R-μ Relationships for 10-Story CH and BH Models and Corresponding SDOF System